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Bootstrap prediction intervals for multivariate time series

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BOOTSTRAP PREDICTION INTERVALS FOR MULTIVARIATE TIME SERIES

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ABSTRACT

The theory and methodology of obtaining bootstrap prediction intervals for univariate time series using the forward representation of the series is extended to vector autoregressive (VAR) models. Kim has shown that simultaneous prediction intervals based on the Bonferroni method and the backward representation of the time series achieve coverage close to nominal when the parameter estimates are corrected for small sample bias. To utilize his method, it is necessary to assume that the innovations are normally distributed to maintain independence of the innovations associated with the backward representation of the time series. This assumption is not necessary if the forward representation is used. Bootstrap prediction intervals based on the forward representation of the time series, are less restrictive and thus can also be adapted for time series that do not have a backward representation.

The asymptotic validity of the proposed bootstrap method is established and small sample properties are studied using Monte Carlo simulation. The simulation study also looks at a number of VAR models including stationary, unit root and near unit root processes. In these models, coverage close to nominal level is reached if the parameter estimates are corrected for small sample bias. In addition to the normal distribution, three non-normal distributions for the innovations are considered, namely the chi-squared, exponential and t distributions. Simulations where prediction intervals are obtained after conducting an order selection of a VAR(2) time series is also studied.

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1. INTRODUCTION

Consider two attributes observed over time, such as closing prices of two different stocks at the New York stock exchange. In most cases, as the price of one stock goes up the other stock tends to increase as well, i.e. the two stock prices are correlated. It is of interest to see where the stock prices will be several days or weeks in the future. Hence, predictors for the stock prices need to be developed. It is straightforward to find a point predictor for time series which gives one possible value of the stock price, say h days ahead. However, a point predictor does not include information about the prediction error. To include this information, prediction intervals, which give an upper and lower bound for the value of each stock price h days ahead, are considered. The bounds are such that the future value of each stock price lies within the corresponding bounds with a certain level of confidence. Many methods have been developed to find prediction intervals for a univariate time series. To be able to gain information jointly about two or more time series, there are, however, many differences compared to the univariate case. In this dissertation, a method for finding prediction regions for multivariate time series utilizing the bootstrap procedure is proposed. The proposed method is applicable to vector autoregressive time series and, due to the use of only the forward representation of the time series, has possible extensions to the vector moving average time series and other more general processes.

Efron (1979) suggested an extension of the jackknife utilizing resampling from a sample to enable statistical inferences with and without the assumption of an underlying distribution of the sample. The main assumption is that each element in the sample is equally likely to be chosen and that the order in which the elements are resampled does not matter. The utilization of the bootstrap method is based on the fact that the bootstrap distribution converges to the true distribution of the underlying sample. It is not possible to apply the method suggested by Efron (1979) directly to time series data. A major feature of time series data is that the observations are correlated. Freedman (1984) suggested estimating the parameters first, and then using the estimated model to estimate the independent and identically distributed (i.i.d.) innovations in stationary linear models. His proposed method then suggests to resample from the estimated innovations to generate a bootstrapped time series utilizing the first observation of the time series as a starting point. He showed that the bootstrapped estimators converge in probability, conditional on the observed data, to the true parameters as the number of observations in the time series goes to infinity. Freedman (1984) notes that the effect of the starting values are negligible for the asymptotic results. The starting values, however, may have an effect on the estimation using bootstrapping in small samples. Efron and Tibshiriani (1986) suggest, like Freedman (1984), to utilize the first p observations in estimation of parameters. Stine (1982) proposes randomly selecting a block of size p of adjacent values.

Thombs and Schucany (1990) use the same technique as Freedman (1984) to obtain the estimates of the innovations. They, however, note that empirical evidence shows that the residuals obtained through estimated model are deflated. Hence, they recommend multipling the centered residuals by a factor suggested by Stine (1987). In addition, Thombs and Schucany (1990) notice that using the first p values of the observed time series to find prediction intervals introduces the problem that the prediction intervals will not be conditional on the last p observations. To obtain prediction intervals having this property they suggest using the backward representation of the time series, where the current value is a function of the future values instead of the past values. Box and Jenkins (1976) suggest the use of a forward difference operator to obtain the time series leading to the backward representation of the time series. With this technique it becomes possible to fix the last p observed values and resample the time series conditional on the last p values. The backward representation, however, introduces the problem that if the true distribution of the innovations is non-Gaussian, then the backward innovations are only uncorrelated and not independent. This violates the assumption of independence needed for the bootstrap. Thombs and Schucany (1990) suggest that if the distribution of the innovations is non-Gaussion, then the forward innovations can be used to find the backward innovations. They show the relationship between the two innovations for the first order autoregressive case. In addition, they show that the bootstrapped distribution converges in distribution to the true distribution of the predicted values, hence establishing the asymptotic vadiltity of the prediction intervals. In the simulation results presented in their paper, it can be seen that the bootstrapped intervals and the prediction interval using the normal approximation given by Box and Jenkins (1976), give average coverage probabilities which are below nominal coverage, hence, they are liberal.

Kreiss and Franke (1992) developed the asymptotic validity of the bootstrapping of M-estimators in case of the univariate ARMA(p, q) case. Extending the idea of bootstrapping the parameters, Pascual, Romo, and Ruiz (2001), and (2004) developed the asymptotic results for bootstrapping prediction intervals for autoregressive integrated moving average (ARIMA) processes. For the ARIMA time series, they assume that the number of unit roots is known enabling the use of a stationary transformation. One major difference from previous papers is that they use the forward representation, since it is not easy to obtain the backward representation for non-autoregressive time series. Differing from previous papers proposing methods for finding prediction intervals using the forward representation, they utilize the last p observations when finding the future values. Hence, the bootstrapped time series is only used to reestimate the parameters. Their simulation results are still liberal, but this is probably due to the bias in the estimators. Kim (1999) discusses the fact that small sample bias may influence the coverage probabilities of prediction intervals and regions. He finds that correcting for bias improves the coverage probabilities, which helps to improve the performance of bootstrapped predication intervals as compared to the standard Box and Jenkins prediction intervals.

For multivariate time series Lütkepohl (1993), provides extensions of prediction intervals proposed by Box and Jenkins. This prediction region is based on the assumption of normality and uses Bonferroni's technique to obtain simultaneous prediction intervals for each component of the time series. In addition, Lütkepohl provides a prediction ellipsoid, which again is based on the assumption that the innovations are normally distributed. As Kim (1999), and (2001) showed in simulation studies, these Bonferroni prediction intervals and the prediction ellipsoid give liberal coverage. Kim (1999), (2001), and (2004) extends the univariate prediction intervals using the backward representation of the time series to the multivariate case. Using bias corrections suggested by Nicholls and Pope (1990) and Kilian (1998), he improves the coverage probabilities such that they are reasonably close to nominal coverage. Kilian suggested a bootstrap-after-bootstrap method for estimation of the unknown parameters by first using the bootstrap to estimate the bias and then using the bootstrap a second time to find the confidence intervals for parameters. In the simulation program provided by Kim (2001), where the properties of the bootstrapped bias correction suggested by Kilian (1998) is examined, Kim does not follow the procedure suggested by Kilian. Details can be found in Sections 3.2. and 6.. Nicholls and Pope (1988) give closed form expression for the bias, which when omitting terms of order higher than $T^{-3/2}$, is usable. Using Nicholls and Pope's (1988) bias correction Kim (2004) notes that these intervals give coverage that is closest to the nominal. Following Kilian (1998), it is desired that the bias correction does not move the parameter estimates into the non-stationary region. Hence, when using the two bias corrections it is necessary to ensure the stationary of the bias corrected estimates. Details can be seen in sections 3.1. and 3.2..

In this dissertation, the bias corrections suggested by Nicholls and Pope (1988) and Killian (1998) are applied to the parameter estimators used in bootstrap prediction regions using forward representation of the time series suggested by Pascual, Romo, and Ruiz (2004) for univariate time series. By using the last p observations when calculating the bootstrapped distribution the prediction regions are conditional on the observed data. In Section 2. the multivariate time series is defined, the least squares estimators are given and general bootstrap procedure is explained. In Section 3., the estimators and the bias correction are explained. Following that Section, methods for finding simultaneous prediction intervals are discussed followed by the section with the proof of the asymptotic validity of the procedure proposed. The description of a simulation study precedes the results and discussion of the simulation study. A final section with concluding remarks and possibilities of future research ends this dissertation.

2. PRELIMINARY RESULTS

2.1. MULTIVARIATE TIME SERIES

First it is necessary to give a definition of a multivariate time series before simultaneous prediction regions can be discussed. In addition, several definitions and results required for the proof of the asymptotic validity of the simultaneous bootstrap prediction intervals for time series are given in this section. Several preliminary definitions necessary to define a vector autoregressive p (VAR(p)) time series is also given. In the following sections, similar notation as given by Lütkepohl (1993) is used.

Definition 2.1 Standard White Noise Process

A sequence of $(K \times 1)$ random variables $U_1,..., U_T$, where $U_t = (U_{1t},..., U_{Kt})'$, is said to be a white noise or innovation process if $E(U_t) = 0$, $E(U_t U_t') = \Sigma_U$ and $E(U_t U_s') = 0$ for $s \neq t$, and for some constant c,

$$E|U_{it}U_{jt}U_{kt}U_{mt}| \leq c \text{ for } i, j, k, m = 1, ..., K \text{ and for all } t.$$

Throughout this dissertation, it will be assumed that the covariance matrix, Σ_u, is non-singular. If the distribution of the U_ts is normal, they will be called Gaussian white noise. Utilizing the definition of a white noise process, it is possible to define the VAR(p) process. The definition is a generalization of the definition of the univariate AR(p) process.

Definition 2.2 Vector Time Series

Let $\nu = (\nu_1,..., \nu_K)'$ be a $(K \times 1)$ vector of intercept terms, A_i, $i = 1,..., p$ be $(K \times K)$ coefficient matrices and U_t be a white noise process. Then a K-dimensional vector valued time series $\{Y_t : t \in \mathbb{Z}\}$, where $Y_t = (Y_{1t},..., Y_{K1})'$, satisfying the difference equation,

$$Y_t = \nu + A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + U_t, \quad t \in \mathbb{Z}, \tag{2.1}$$

is said to be a VAR(p) process.

Let M_i, $i = 1,..., q$ be $(K \times K)$ coefficient matrices and U_t be a white noise process. Then a vector time series $\{Y - t : t \in \mathbb{Z}\}$ satisfying,

$$Y_t = U_t + M_1 U_{t-1} + ... + M_q U_{t-q}, \quad t \in \mathbb{Z}, \tag{2.2}$$

is said to be a vector moving average process.

Combining the VAR(p) and vector MA(q) results in a VARMA(p, q) time series $\{Y - t : t \in \mathbb{Z}\}$ that satisfies

$$Y_t = \nu + A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + U_t + M_1 U_{t-1} + ... + M_q U_{t-q}, \quad t \in Z. \tag{2.3}$$

In this dissertation, only VAR(p) time series will be considered. It is assumed that the coefficients $\nu, A_1,..., A_p$ and Σ_U are unknown and thus must be estimated. Most of the theory in time series analysis relies on the possibility of representing

Y_t as an infinite vector moving average time series. To be able to do this, the time series must satisfy the following condition.

Definition 2.3 Stable time series

A time series Y_t given in equation (2.1) is said to be stable if,

$$det(I_K - A_1 z - ... - A_p z^p) \neq 0 \qquad \text{for } |z| \leq 1.$$

The property of a time series being stable is also referred to as the time series being stationary. Just as in the univariate case (See Fuller (1996) p. 59), it is possible to represent the forward VAR(p) as

$$\mathbf{W}_t = \Pi \mathbf{W}_{t-1} + \mathbf{U}_t, \tag{2.4}$$

where $\mathbf{W}_t = (Y_t', ..., Y_{t-p+1}')'$ and $\mathbf{U}_t = (U_t', 0, ..., 0)'$ are $Kp \times 1$ vectors and the $Kp \times Kp$ matrix Π is given as

$$\Pi = \begin{bmatrix} A_1 & \dots & A_p \\ & I_{Kp-K} & \mathbf{0} \end{bmatrix}.$$

This representation is helpful in proving the asymptotic results of the bootstrap method.

A possible way of utilizing the bootstrap for prediction intervals is by rewriting the time series given in equation (2.1) as a difference equation of future observations.

This backward model is used to generate bootstrap time series conditioned on the last p observations, $Y_{T-p+1},..., Y_T$. Kim (1998) provides the following result.

Proposition 2.4 Backward representation of VAR(p)

Let $\{Y_t : t \in \mathbb{Z}\}$ be a time series satisfying the difference equation (2.1). Then the time series satisfies the difference equation given by,

$$Y_t = \mu + H_1 Y_{t+1} + \cdots + H_p Y_{t+p} + V_t, \tag{2.5}$$

where $\mu = (\mu_1,..., \mu_K)'$ is a $(K \times 1)$ vector of intercept terms, the coefficient matrices H_i are $(K \times K)$ matrices and the v_t are white noise with mean zero and nonsingular covariance matrix Σ_V.

Proof: Can be found in Kim (1998).

The innovations, V_t, for the backward representation of the time series are only uncorrelated and not independent if the U_ts are not Gaussian. However, as seen in the simulation study done by Kim (2001) and (2004), this does not negatively affect the coverage probabilities of the prediction intervals. Similarly, as with the forward representation, the backward representation can be written as,

$$\mathbf{W}_t = \Omega \mathbf{W}_{t+1} + \mathbf{V}_t, \tag{2.6}$$

where $\mathbf{V}_t = (0,...,0,V_t')'$ is a $Kp \times 1$ vectors and the $Kp\times Kp$ matrix Ω is given as

$$\Omega = \begin{bmatrix} \mathbf{0} & I_{Kp-K} & \\ H_1 & \dots & H_p \end{bmatrix}.$$

As stated by Kim (2001), Π and Ω are related as $\Pi = \Gamma\Omega'\Gamma^{-1}$, where $\Gamma = E(\mathbf{W}_t\mathbf{W}_t')$.

2.2. DEFINITIONS AND RESULTS OF CONVERGENCE

To be able to discuss the consistency of the least squares estimators and asymptotic validity of the bootstrap method, it is necessary to define different types of convergence. In addition, several results from probability theory are helpful when proving the asymptotic validity of the proposed method. These results are also stated in this section.

Definition 2.5 Weak Convergence

Let X and X_n be random variables with distribution functions F_X and F_{X_n} for all $n \in \mathbb{N}$. Then, it is said that X_n converges weakly to X if,

$$\lim_{n\to\infty} F_{X_n}(x) = F_X(x),$$

for each x at which F is continuous.

Definition 2.6 Weak Convergence in Probability

Let X_n be a sequence of random variables with distribution functions, $F_{X_n}(x)$ and let X have a distribution function $F_X(x)$. Then, X_n converges weakly in probability to X if,

$$P\Big(\big|F_{X_n}(x) - F_X(x)\big| \geq \varepsilon\Big) \longrightarrow 0 \qquad \text{as } n \to \infty.$$

for all $\varepsilon > 0$ and for all x except on a set of probability zero.

Definition 2.7 Convergence in Probability

Let X_n be a sequence of random variables. Then, X_n converges to X in probability if,

$$P\Big(\big|X_n - X\big| \geq \varepsilon\Big) \longrightarrow 0 \qquad \text{as } n \to \infty.$$

Definition 2.8 Convergence Almost Everywhere (a.e.)

Let X_n be a sequence of random variables. Then, X_n converges to X almost everywhere if,

$$P\Big(\big|X_n - X\big| \leq \varepsilon\Big) \longrightarrow 1 \qquad \text{as } n \to \infty.$$

Convergence in probability implies weak convergence. This result will be utilized in the proof of the asymptotic validity of the simultaneous bootstrap prediction interval. Lütkepohl (1993) denotes convergence in probability, as defined in Definition 2.7, by plim. This notation will be used in the following sections.

Definition 2.9 Weak Convergence Almost Everywhere

Let X_n be a sequence of random variables with distribution functions $F_{X_n}(x)$ and let X have a distribution function $F(x)$. Then X_n converges weakly almost everywhere to X if, for all x except on a set of probability zero,

$$P\Big(\big|F_{X_n}(x) - F_X(x)\big| = 0\Big) \quad \longrightarrow \quad 1 \qquad \text{as } n \to \infty.$$

Theorem 2.10 Slutsky's Theorem

Let X_n and Y_n be sequences of random variables that converge weakly to X and Y respectively. Then $X_n Y_n$ converges weakly to XY.

Proof: Can be found in Fuller (1996), Corollary 5.2.6.1.

Pascual, Romo, and Ruiz (2004) mention that there exists a bootstrap version of Slutsky's theorem that gives a stronger result. In this case weak convergence is replaced with weak convergence in probability. It was, however, not possible to find the reference in which this result is stated. If this result is true, it is possible to extend the proof of the theoretical result proven in Section 5.3., such that the convergence is not just weak, but weak convergence in probability.

Theorem 2.11

Let X_n be a sequence of random variables that converges in probability to c and let $g(x)$ be a continuous function. Then $g(X_n)$ converges in probability to $g(c)$.

Proof: Can be found in Fuller (1996), Corollary 5.2.1.2.

Since a distribution function is often not assumed when using the bootstrap, it is necessary to find a substitute. The empirical distribution function based on the data is used to estimate the true distribution function without the assumption of a certain parametric model.

Definition 2.12 Empirical Distribution Function

Let $y_1,\ldots,y_n$ be a sample from i.i.d. random variables $Y_1,\ldots Y_n$ with PDF f and CDF F. The empirical distribution function (EDF) $\hat{F}$ is defined as,

$$\hat{F}(y) = \frac{\#\{y_j \le y\}}{n} = \frac{1}{n}\sum_{i=1}^{n} I_{(-\infty,\, y_i]}(y),$$

where $\#\{A\}$ denotes the number of times the event A occurs and $I_A(y)$ denotes the indicator function of the set A, hence equals 1 if $y \in A$ and 0 otherwise.

For a given random sample, the empirical distribution function has the useful property of converging to the true distribution function. Data from a time series, as given in Definition 2.2, does not satisfy the assumption of independence that occurs with a random sample. This makes the bootstrap results more challenging to prove. In addition, it is not always feasible to find the exact bootstrap distribution. In this case, a Monte Carlo approximation is used to estimate the bootstrap distribution. A random sample from the true bootstrap distribution is taken and the empirical distribution function is used to estimate the true bootstrap distribution function. As described in Section 5.2., this method is used when actually calculating simultaneous bootstrap prediction intervals in the simulation study. The asymptotic validity of this method is based on the following theorem.

Theorem 2.13

Let $Y_1,\ldots,Y_n$ be a random sample with distribution function F. Then the empirical distribution function, $\hat{F}(y)$, based on the random sample, converges to $F(y)$ for all $y \in \mathbb{R}$ as $n \to \infty$.

Proof: Follows from Glivenko-Cantelli Theorem, which can be found in Billingsley (1995), Theorem 20.6.

2.3. MULTIVARIATE LEAST SQUARES ESTIMATOR

Similar to the univariate case, several methods are available to estimate the unknown parameters in multivariate time series models. In this dissertation, least squares estimators are used for the VAR(p) time series, since the asymptotic properties of the bootstrap prediction intervals can be proven easily if the least squares estimators are used. It is also possible to find estimators for VARMA(p, q) time series, however, since prediction regions for these time series are not considered in this dissertation, these are not considered. Methods for finding estimators for VARMA time series can be found in Lütkepohl (1993) and Mitchell and Brockwell (1997).

Lütkepohl (1993) derives the least squares estimator for ν, $A_1,\ldots, A_p$ in the following manner. Given the observed time series $Y_1, \ldots, Y_T$ and initial values $Y_{1-p}, \ldots Y_0$, define,

$$
\begin{aligned}
Y &= (Y_1, \ldots, Y_T)_{(K \times T)} \\
A &= (\nu, A_1, \ldots, A_p)_{(K \times (Kp+1))} \\
Z_t &= \begin{bmatrix} 1 \\ Y_t \\ \vdots \\ Y_{t-p+1} \end{bmatrix}_{((Kp+1)\times 1)} \\
Z &= (Z_0, \ldots, Z_{T-1})_{((Kp+1)} \times T) \\
U &= (U_1, \ldots, U_T)_{(K \times T)}.
\end{aligned} \tag{2.7}
$$

Theorem 2.14 Least Squares Estimator

Let $Y_1, \ldots, Y_T$ be a sequence of random variables and let $Y_{1-p}, \ldots, Y_0$ be initial values that satisfy the difference equation (2.1). Utilizing the notation given in equation (2.7), then the least squares estimator for A is given by,

$$
\hat{A} = YZ'(ZZ')^{-1} = A + UZ'(ZZ')^{-1}. \tag{2.8}
$$

Proof: The proof can be found in Lütkepohl (1993) p. 63-65.

Note that prior to observing the data, $\hat{A}$ is a random variable, hence an estimator; and after the time series is observed $\hat{A}$ is a specific value, hence an estimate. In the following, the same notation is used for the estimator as well as the estimate. It is noted that in the theoretical derivation, it is assumed that the estimator is used, whereas in the simulation as well as practical applications the estimate is used. When using the backward representation of the time series, estimators for $\mu, H_1,..., H_p$, are needed. Reversing the order of the time series, hence writing the last element first and so on, the equation (2.8) can be utilized to estimate $\mu, H_1,..., H_p$.

Kim (2001) uses the representation given in equations (2.4) and (2.6) to find the least squares estimators of Π and Ω. His results are stated in the following theorem.

Theorem 2.15

Let $Y_1,..., Y_T$ be a sequence of random variables and let $Y_{1-p},..., Y_0$ be initial values that satisfy the difference equation (2.1). Writing the time series in the state space representation given in equations (2.4) and (2.6), respectively, the least squares estimators for Π and Ω are given by,

$$\hat{\Pi} = \left(\sum \mathbf{W}_t \mathbf{W}'_{t-1}\right)\left(\sum \mathbf{W}_{t-1}\mathbf{W}'_{t-1}\right)^{-1} = \Pi + w_T C_T^{-1}, \text{ and} \tag{2.9}$$

$$\hat{\Omega} = \left(\sum \mathbf{W}_t \mathbf{W}'_{t+1}\right)\left(\sum \mathbf{W}_{t+1}\mathbf{W}'_{t+1}\right)^{-1} = \Omega + \delta_T D_T^{-1}, \tag{2.10}$$

where $w_T = 1/T \sum U_t \mathbf{W}'_{t-1}$, $C_T = 1/T \sum \mathbf{W}_{t-1}\mathbf{W}'_{t-1}$, $\delta_T = 1/T \sum V_t \mathbf{W}'_{t+1}$ and $D_n = 1/T \sum \mathbf{W}_{t+1}\mathbf{W}'_{t+1}$.

Proof: The proof follows the proof of Theorem 2.14.

Kim (2001) notes that the relationship between the least squares estimators $\hat{A}$ and $\hat{\Pi}$ is such that

$$\hat{\Pi} = \begin{bmatrix} \hat{A}_1 & \cdots & \hat{A}_p \\ I_{K(p-1)} & & 0 \end{bmatrix}.$$

Hence results for $\hat{\Pi}$ are applicable for results of $\hat{A}$.

2.4. ASYMPTOTIC PROPERTIES OF THE LEAST SQUARES ESTIMATORS

Lütkepohl (1993) states the asymptotic properties for the least squares estimators given in Theorem 2.14. He notes that the small sample properties of the least squares estimators are challenging to derive. Even most of the asymptotic results require that the innovations are a standard white noise process, as defined in Definition 2.1. If the U_t are normally distributed (Gaussian), this is the case. The following Lemma follows Lütkepohl's (1993) Lemma 3.1 on page 66.

Lemma 2.16

Let $Y_1,\ldots,Y_T$ be a sequence of random variables and let $Y_{1-p},\ldots,Y_0$ be initial values that satisfy the difference equation (2.1). In addition let U_t, $t \in \mathbb{Z}$, be a sequence of standard white noise innovations. Then:

1. $\Gamma = \lim_{T\to\infty} ZZ'/T$ exists and is nonsingular, and

2. $\frac{1}{\sqrt{T}}(Z \otimes I_K)\mathbf{U}$ converges in distribution to a random variable, with $N(0, \Gamma \otimes \Sigma_u)$,

where $\otimes$ is the direct product of two matrices.

Proof: Similar to Theorem 8.2.3 of Fuller (1976 p. 340)

The above two properties are assumptions necessary to prove the asymptotic properties of the least squares estimator, given by Lütkepohl (1993).

Proposition 2.17

Under the assumptions of Lemma 2.16, the least squares estimator $\hat{A} = YZ(ZZ')^{-1}$ for the coefficient matrix A as defined in equation (2.7) has the following properties,

1. $\hat{A}$ converges in probability to A, and

2. $\sqrt{T}\text{vec}(\hat{A}-A)$ converges in distribution to a random variable with $N(0, \Gamma^{-1} \otimes \Sigma_u)$, where $\Gamma = \lim_{T\to\infty} ZZ'/T$.

Proof: Can be found in Lütkepohl (1993) p.66-67

It is necessary to know Γ and Σ_U to assess the asymptotic dispersion of $\hat{A}$. Since Γ and Σ_U are not known, they need to be estimated. Lütkepohl (1993) suggests using,

$$\begin{aligned} \hat{\Gamma} &= ZZ'/T, \text{ and} \\ \tilde{\Sigma}_u &= \frac{1}{T}Y(I_T - Z'(ZZ')^{(-1)}Z)Y' = \frac{1}{T}\sum_{t=1}^{T} \hat{u}_t\hat{u}_t{}', \end{aligned}$$

where Z and Y are defined in equation (2.7). $\tilde{\Sigma}_U$ is not an unbiased estimator which can, however, be corrected by multiplying by $T/(T - Kp - 1)$, instead of by $1/T$. This estimator will be denoted by $\hat{\Sigma}_U$. It can be shown that both estimators are asymptotically equivalent. Lütkepohl also shows that $\hat{\Sigma}_u$ has the same asymptotic properties as the estimator,

$$\frac{UU'}{T} = \frac{1}{T}\sum_{t=1}^{T} u_t u_t',$$

which is based on the true innovations, which are unknown, making this estimator not feasible to use. Consistency of these least squares estimators, given in equation (2.9) and (2.10), follows from Freedman (1984). He shows that $\operatorname{plim} w_T = \operatorname{plim} \delta_T = 0$ and $\operatorname{plim} F_t = \operatorname{plim} D_T = \Gamma$.

2.5. GENERAL DEFINITIONS FOR THE BOOTSTRAP

For most statistical procedures it is necessary either to assume a certain parametric model, or to use the fact that the statistic considered is asymptotically normally distributed. However, given a sample, it may not be possible to determine the correct distribution from which data originated, or the sample size is too small or too skewed to be able to use the normal approximation. In many of these cases, bootstrap methods help provide approximations for the statistics of interest. This section gives a general overview of methods available for bootstrapping.

Given a sample from a population, say $y_1,\ldots,y_n$, it is of interest to make statistical inferences about a population characteristic, θ. The sample values are all outcomes of independent and identically distributed (i.i.d.) random variables, $Y_1,\ldots,Y_n$, whose probability density function (pdf) and cumulative distribution function (CDF) are denoted by f and F, respectively. In the following sections, it is assumed that the statistic or its approximation used to estimate θ has been chosen appropriately. Unlike in the parametric model, where a specific distribution function is assumed, no specific distribution function is assumed for the bootstrap prediction regions proposed in this dissertation.

When no distribution function for the observed data is assumed, the EDF, as defined in Definition 2.12, plays the role of the CDF with estimates substituted for unknown parameters in the parametric case. Note that the mean of the EDF is the sample mean, $\bar{y} = n^{-1} \sum y_i$. Following Davison and Hinkley (2003), the statistic, t, used to estimate θ will not depend on the order of the sample, $y_1,...y_n$, and hence can be expressed as $t = t(\hat{F})$, where $t(\cdot)$ is a statistical function, which is essentially just a mathematical expression of the algorithm for computing t from $\hat{F}$. However, not all estimators of interest can be expressed in this way. To be more formal, it would be necessary to write $T = t_n(\hat{F})$. In the cases considered here, it does not make a difference. It is necessary to define the parameter and the estimator in a robust way, without making any assumptions about the distribution function, F. This is done by using the representation, $\theta = t(F)$, which is sometimes called robustness of specification, noted by Davison and Hinkley (2003).

In many statistical applications approximate distributions need to be found. For example, to be able to find prediction intervals it is required to have some estimate of an upper and lower prediction bound, which can be accomplished using the normal distribution as an approximation. In addition, in some cases, even if appropriate distributions are found, not all parameters are known. Bootstrap methods can help estimate these parameters. However there are additional methods that do not require the assumption of a parametric model.

If an assumed distribution is fitted by substituting unknown parameters by their appropriate estimates, and the resamples are generated from this distribution, the procedure is called parametric bootstrap, as described by Davison and Hinkley (2003). The methods described in this dissertation utilize the bootstrap without assuming a certain distribution function. This is often called nonparametric bootstrapping, where the samples are taken from the empirical distribution function. An "$*$" will be used to denote these bootstrapped quantities. The estimated quantities can be the actual value, Y^*, or a statistic such as T^*, or a percentile point of the distribution. In some cases theoretical calculations are too complex to find properties of the statistic T. Hence, the bootstrap enables the estimation of those quantities. Efron (1979) gives the steps for the general bootstrap.

1. Construct the empirical distribution.
2. Resample B times n replicates from the empirical distribution and calculate the statistic of interest, T^*.
3. Use the B replicates of T^* to estimate the properties of the statistic, T.

The statistic of interest can be the variance and bias, which then can be used in a prediction interval using the normal approximation. When using the normal approximation the variance is estimated, so the statistic used in finding prediction bounds has a t-distribution. Hence, these prediction intervals are denoted as percentile-t prediction intervals. In some cases, however, it may not be appropriate to use the normal approximation. In this case, it is possible to utilize the B replicates of T^* to estimate the percentile points of the ture distribution function. These intervals are called percentile intervals. In fact, one way of finding interval estimates is that the B replicates of T give an empirical distribution of the random variable T. Hence, percentiles of this bootstrapped distribution can be directly used to estimate the upper and lower bound. Section 5.3. will state when the bootstrap procedure is consistent, i.e. asymptotically valid.

3. BIAS CORRECTION FOR THE LEAST SQUARES ESTIMATORS

Kim (2001) noted that the least squares estimators have considerable bias leading to bootstrap prediction intervals and regions that are too liberal. Kim (2001) and (2004) proposed the use of two methods for correcting the bias when calculating bootstrap prediction intervals. The first, used by Kim (2001), is based on a bootstrap-after-bootstrapping method proposed by Kilian (1998), where an initial bootstrap is used to estimate the bias. Kim (2004) proposed the use of the bias correction derived by Nicholls and Pope (1988) and extended it to the case of the backward representation of the VAR(p) time series. The two methods will be described in detail in the next two subsections.

3.1. BIAS CORRECTION PROPOSED BY NICHOLLS AND POPE (1988)

In their 1988 paper, Nicholls and Pope propose a method of correcting for the bias in the least squares estimators of a multivariate autoregressive model. They obtain an explicit expression, which, as they suggest, is useful when it is necessary to make a bias correction prior to bootstrapping. Given the following model using the same notation as above, $\mathbf{W}_t = \Pi\mathbf{W}_{t-1} + \mathbf{U}_t$, where $\mathbf{Y}_t$ and $\mathbf{U}_t$ are $Kp \times 1$ vectors, and Π is a $(Kp \times Kp)$ matrix. Note that the innovations U_T on which $\mathbf{U}_t$'s are based, are i.i.d. with mean zero and covariance matrix Σ_U. The following assumptions, stated in Nicholls and Pope (1988), are necessary for the model given in equation (2.4):

A1 The model is stationary, hence all roots of A are within the unit circle,

A2 for all $s > 0$, $E||U_t||^s < \infty$, and

A3 as $n \to \infty$, $E\left(||C_n(0)^{-1}||^2\right) = O(1)$,

where, for $s = -1, 0$,

$$C_n(s) = \frac{1}{n-1} \sum_{i=1}^{n-1} X_{i-s} X_i^T,$$

where $X_i = Y_i - \bar{Y}_n$. Nicholls and Pope note that (A2) and (A3) hold when the innovations, U_t, are normally distributed. Under these assumptions, they prove the following theorem:

Theorem 3.1

Let $\hat{\Pi}$ be the least squares estimator of the coefficient matrix Π of the model (2.4), based on a sample size, T. Suppose the innovations, $\mathbf{U}_t$, of (2.4) have U_t's which are Gaussian with variance matrix Σ_U. Then the bias is given as

$$B_\Pi = E(\hat{\Pi}) - \Pi = -\frac{b_\Pi}{T} + O(T^{-3/2})$$

where

$$
\begin{aligned}
b_{\Pi} &= \Sigma_U \Bigg[(I_{Kp} - \Pi')^{-1} + \Pi'(I_{Kp} - \Pi'^2)^{-1} \\
&\quad + \sum_{\lambda \in Spec(\Pi')} \lambda(I_{Kp} - \lambda\Pi')^{-1} \Bigg] \Gamma(0)^{-1}, \qquad (3.1)
\end{aligned}
$$

with $\Gamma(0) = \mathbb{E}[\mathbf{W}_t \mathbf{W}_\mathbf{t}]$ and $Spec(\Pi)$ denoting the set of all eigenvalues of Π.

Proof: See Nicholls and Pope (1988).

This theorem was extended by Kim (2004) to the case of the backward representation of the time series. Following the model given in equation (2.6), the bias for Ω is given by,

$$
B_{\Omega} = \mathbb{E}(\hat{\Omega}) - \Omega = -\frac{b_{\Omega}}{T} + O(T^{-3/2}),
$$

where,

$$
b_{\Omega} = \Sigma_V \left[(I - \Omega')^{-1} + \Omega'(I - \Omega'^2)^{-1} + \sum_{\lambda \in Spec(\Omega')} \lambda(I - \lambda\Omega')^{-1} \right] \Gamma(0)^{-1}. (3.2)
$$

In most cases Σ_U, Σ_V and $\Gamma(0)$ are not known, and Kim (2004) proposes to replace them with their least square estimators given by,

$$\hat{\Sigma}_U = \frac{1}{T-Kp-1}\sum_{i=1}^{T}\hat{U}_t\hat{U}_t' = \frac{1}{T}Y(I_T - Z'(ZZ')^{-1}Z)Y', \tag{3.3}$$

$$\hat{\Sigma}_V = \frac{1}{T-kp-1}\sum_{i=1}^{T}\hat{U}_t\hat{U}_t' = \frac{1}{T}Y_b(I_T - Z_b'(Z_bZ_b')^{-1}Z_b)Y_b', \text{ and} \tag{3.4}$$

$$\hat{\Gamma}(0) = \frac{1}{T}ZZ'. \tag{3.5}$$

These estimators are substituted in equations (3.1) and (3.2) and terms of order $T^{-3/2}$ and higher are omitted. Note that Z_b and Y_b are defined equivalently as Z and Y using the backward representation of the time series. It follows directly that the bias correction of Nicholls and Pope (1988) converges in probability to zero.

3.2. BIAS CORRECTION PROPOSED BY KILIAN (1998)

Kilian (1998) proposed a method to adjust for small sample bias when finding confidence intervals for the orthogonalized impulse responses. Only his bias correction is considered in this dissertation. Details about orthogonalized impulse responses can be found in Kilian (1998). He made clear, that even though the asymptotic distribution is normal, the small-sample distributions may be extremely biased or skewed. Since it is difficult to eliminate the bias for the orthogonalized impulse responses, and the small-sample bias of the ordinary least squares estimators is responsible for the bias of the orthogonalized impulse responses, he suggests correcting the bias prior to bootstrapping the estimates for the orthogonalized impulse responses. Since, when finding prediction intervals only the least squares estimators and their bias are of interest, this method is applicable to the problem discussed in this dissertation. The method proposed by Kilian (1998) is described in detail below.

1. Estimate the coefficients of the VAR(p) model and estimate the innovations, U_t^*, replacing A with its least squares estimators in equation (2.1) and solving for U_t.

2. Generate B bootstrap replications of the time series $Y_1,...,Y_T$ utilizing the difference equation,

$$Y_t^* = \hat{\nu} + \hat{A}_1 Y_{t-1}^* + ... + \hat{A}_p Y_{t-p}^* + U_t^*, \tag{3.6}$$

   using standard bootstrap techniques, which are described in more detail in Section 2.5.. For each replication, estimate the coefficient matrix A, denoted by A_i^*, $i = 1,...,B$. The bias $\Psi = \mathbb{E}[\hat{A} - A]$ is approximated by $\hat{\Psi} = \bar{\hat{A}}^* - \hat{A}$, where $\bar{\hat{A}}^*$ is the average overall bootstrapped estimators from equation (3.6).

To ensure that by doing the bias correction, the estimator does not fall out of the region where the time series is stationary, the following adjustment is necessary when correcting for the bias.

3. Calculate the absolute value of the largest eigenvalue of the matrix

$$\begin{bmatrix} \hat{A}_1 & \dots & \hat{A}_p \\ I_{K(p-1)} & 0 & \end{bmatrix},$$

where $\hat{A}_i$ are the least squares estimators of A and following Kilian, denote this quantity by $m(\hat{A})$. If $m(\hat{A}) \geq 1$, implying the roots are outside the unit circle, set $\tilde{A} = \hat{A}$ without any adjustments. If $m(\hat{A}) < 1$, set $\tilde{A} = \hat{A} - \hat{\Psi}$ and calculate $m(\tilde{A})$. If $m(\tilde{A}) \geq 1$, let $\hat{\Psi}_1 = \hat{\Psi}$ and $\delta_1 = 1$ and define the following iteration:

a) let $\hat{\Psi}_{i+1} = \delta_i \hat{\Psi}_i$ and $\delta_{i+1} = \delta_i - 0.01$, $\tilde{A}_i = \hat{A} - \hat{\Psi}_i$,

b) when $m(\tilde{A}_i) < 1$ set $\tilde{A} = \tilde{A}_i$.

Following the notation given by Kim, set $\tilde{A}^c = \tilde{A}$.

As Kilian (1998) noted, the grid used in step (3a) and (3b) avoids pushing a stationary estimator of the parameters into the non-stationary region. In addition, he notes that "the adjustment has no effect asymptotically and does not restrict the parameter space of the OLS, since it does not shrink the OLS estimate $\hat{A}$ itself, but only its bias estimate." Since the least squares estimator, $\hat{A}$, and the bootstrapped estimator, $\bar{\hat{A}}^*$, converge in probability to the true parameter matrix, A, it follows that the difference, $\bar{\hat{A}}^* - \hat{A}$, converges to zero in probability. Thus, both bootstrap bias corrected estimators converge in probability to the true parameter, A.

When finding the bootstrap prediction intervals it will be necessary to use the estimator, $\tilde{A}^c$, to generate the bootstrapped time series. From this time series the least squares estimator is calculated and corrected for bias using the procedure described. This estimator will be denoted by $\tilde{A}^{*c}$. However, using the same scheme and assuming in each bootstrap step 1000 replications are used, the amount of computations would be $(1000 + 1000 \times 1000)$. Kilian suggests using the $\hat{\Psi}$ found in step (2) as an estimate of the bias estimate $\hat{\Psi}^*$. As Kilian (1998) notes, $\hat{\Psi}$ agrees up to an order $O(T^{-3/2})$ with $\hat{\Psi}^*$ and that $\hat{\Psi}$ is of order $O_P(T^{-1})$ itself. A flowchart of the implementation of the bias correction is given in Appendix A.

4. PREDICTION REGIONS

4.1. INTRODUCTION

Different methods have been discussed to find prediction intervals for both univariate and multivariate time series. Lütkepohl discusses asymptotic prediction intervals based on the assumption of normal innovations. However simulation studies have shown that these prediction intervals are liberal. In an attempt to make the coverage closer to nominal coverage, several methods have been proposed. Thombs and Schucany (1990) discuss bootstrap prediction intervals for univariate AR(p) time series, but due to small sample bias in the estimators, these prediction intervals are also liberal. Kim (2001) suggests correcting the bootstrap prediction intervals for their bias, using bootstrap-after-bootstrap to estimate the bias proposed by Kilian (1998). In (2004), Kim suggested using a method proposed by Pope (1988) to adjust the estimators for bias. This method has the benefit of not being as computationally intensive as the bootstrap-after-bootstrap, since here the bias is calculated without needing to generate bootstrap replications. However, Kim only applies both these corrections to the backward representation of the time series. Pascual, Romo, and Ruiz (2004) suggest using a forward representation of the time series when finding prediction intervals for a univariate time series. In this method, the estimated parameters are not conditional on the last p observations, however, the bootstrapped future values are conditional on the last p observations. The benefit of this method is that it does not require the backward representation of the time series, hence this method can be more easily applied to MA and ARMA processes. In addition, Pascual, Romo, and Ruiz apply the method to an ARIMA(p, d, q) process, under the assumption that d is known. Their use of the forward representation differs from the method proposed by Masarotto (1990) and Grigoletto (1998), where the future values are not conditioned on the last p values. In this dissertation, an extension of the method of Pascual, Romo, and Ruiz to VAR(p) time series is considered. In a simulation study it is shown that this method is comparable, with respect to coverage probabilities, to the method suggested by Kim (2001) and (2004). The proposed method, however, does not require the backward representation of the time series. In addition, the asymptotic validity of the proposed method is shown in Section 5.3.

4.2. ASYMPTOTIC PREDICTION INTERVALS

Let $\{Y - t : t \in \mathbb{Z}\}$ be a time series, where $Y_t = (Y_{1t}, \dots, Y_{Kt})'$, satisfying

$$Y_t = \nu + A_1 Y_{t-1} + A_p Y_{t-p} + U_t, \tag{4.1}$$

where $\nu = (\nu_1, \dots, \nu_K)'$ is a $(K \times 1)$ vector of intercept terms, the coefficient matrices A_i are $(K \times K)$ matrices and the U_t are white noise with mean zero and nonsingular covariance matrix Σ_U. The coefficient matrices, A_i, the covariance matrix of the white noise, Σ_U, and intercept ν are assumed unknown. Utilizing the least squares estimators as given in Theorem 2.14, a point predictor is given as,

$$\hat{Y}_T(h) = \hat{\nu} + \hat{A}_1\hat{Y}_T(h-1) + ... + \hat{A}_p\hat{Y}_T(h-p), \tag{4.2}$$

where $\hat{\nu}$ and $\hat{A}_i$ are the elements of $\hat{A} = (\hat{\nu},\ \hat{A}_1,..., \hat{A}_p)$ of the least squares estimators, found in Theorem 2.14, of the coefficient matrix A. Note that $\hat{Y}_T(h-i) = y_{T+h-i}$ for $h - i \leq 0$. Using a point predictor does not give any information about the prediction error. To take this into account, a prediction interval for Y_{T+h} is being considered. Following Brockwell and Davis (1995) a formal definition of a prediction interval is as follows.

Definition 4.1 Prediction Interval

Let $y_1,..., y_T$ be a realization of the time series following the difference equation given in equation (4.1). Then an interval $[L_k(h), U_k(h)]$ is called a $(1-\alpha)$-prediction interval for $Y_{k,T+h}$, where $Y_{k,T+h}$ is the k-th element of the vector Y_{t+h}, if

$$P(L_k(h) \leq Y_{k,T+h} \leq U_k(h)) = 1 - \alpha. \tag{4.3}$$

Often the bounds are found using past observations, hence the bounds are functions of $y_1,..., y_T$. Since the parameter matrices are not known, it is not always possible to find bounds satisfying equation (4.3). In this case, it may be possible to find prediction bounds, which satisfy equation (4.3) asymptotically. It will be shown in Section 5.3. that the bootstrap prediction bounds discussed in this dissertation satisfy equation (4.3) asymptotically.

Under the assumptions that the coefficients and the variance-covariance matrix of innovations are known and $U_t \sim N(0, \Sigma_u)$, it is possible to derive prediction intervals, which satisfy equation (4.3). Following Lütkepohl (1993), it is noted that,

$$Y_{T+h} - Y_T(h) = \sum_{i=0}^{h-1} A_i U_{t+h-i} \quad \sim \quad N(0, \Sigma_Y(h)).$$

Using the properties of multivariate normal random variables implies that,

$$\frac{Y_{k,T+h} - Y_{k,T}(h)}{\sigma_k(h)} \quad \sim \quad N(0,1),$$

where $Y_{k,T}(h)$ is the k-th component of $Y_T(h)$ and $\sigma_k(h)$ is the square root of the k-th diagonal element of $\Sigma_Y(h)$, which is found using equations (4.4) through (4.7). Letting z_α denote the $(1-\alpha)100$-th percentile of the normal distribution, a $(1-\tau)100\%$ h-step ahead prediction interval is given as,

$$Y_{k,T}(h) \pm z_{\tau/2}\sigma_k(h).$$

Utilizing Bonferroni's method, choosing $\tau = \alpha/K$, simultaneous prediction intervals with coverage of at least $(1-\alpha)$ are achieved for Y_{T+h}. Since Bonferroni's method usually gives very conservative intervals, it would be possible to utilize the fact that,

$$[Y_{T+h} - Y_T(h)]'F'(F\Sigma_Y(h)F')^{-1}F[Y_{T+h} - Y_T(h)] \quad \sim \quad \chi^2(N),$$

where the $(N \times K)$ matrix F is defined as the matrix $[I_T, 0]$.

Since in most cases neither the coefficient matrices nor the variance-covariance matrix of the innovations are known, the previously described methods need to be adapted to this case. For the Bonferroni interval, estimators are substituted for all the unknowns. Kim (2001) notes that under the assumption of normal innovations,

$$\left[Y_{T+h} - \hat{Y}_T(h)\right]'\hat{\Sigma}_Y(h)^{-1}\left[Y_{T+h} - \hat{Y}_T(h)\right] \xrightarrow{T\to\infty} \chi^2(K),$$

where $\hat{\Sigma}_Y(h)$ is the asymptotic mean square error matrix of $\hat{Y}_T(h)$, as given by Lütkepohl (1993).

4.3. BOOTSTRAP PREDICTION INTERVALS

One of the possible benefits of considering bootstrap prediction intervals is that no assumption of the distribution is necessary. As seen in the simulation studies done by Kim (2001) and (2004) and Pascual, Romo, and Ruiz (2004), bootstrap prediction intervals can be used even if the assumption of normality is violated. The prediction intervals suggested for the univariate case by Pascual, Romo, and Ruiz (2004) are extended to the multivariate case and shown to have similar coverage probabilities as the bootstrap prediction intervals suggested by Kim (2001) and (2004). The two types of bootstrap prediction intervals considered here are based on the percentile method and the percentile t-method. However, without correcting for the bias in estimates of the coefficient matrices, the bootstrap prediction intervals are liberal. Kim (2001) and (2004) suggests two methods to correct for the small sample bias. The first one is based on a bootstrapped estimator for the bias (see Section 3.2.

for details), whereas the second is based on the theoretical bias, omitting terms of order higher than $O(n^{(3/2)})$, which was suggested by Nicholls and Pope (1988) as described in Section 3.1..

In the following sections, it is assumed that the observed time series, $y_1,..., y_T$, follows the difference equation as defined in equation (4.1), where A and Σ_U are assumed to be unknown.

4.3.1. Bootstrap Prediction Intervals Based on the Backward Representation of the Time Series. Thombs and Schucany (1990) suggest using the backward representation of the time series given in equation (4.1) when finding prediction intervals for a time series. The benefit is that the resampled time series are conditional on the last p observed values, since when using the backward representation given in Proposition 2.4 it is possible to fix the last p observations and generate the first $T - p$ observations. Also, using bias corrections for the estimated parameters, Kim (2001) and (2004) suggested using the following procedure to find prediction intervals:

1. Given $(y_1,..., y_T)$, estimate the coefficient matrices $A = (\nu, A_1,..., A_p)$ and $H = (\mu, H_1,..., H_p)$ using least squares estimation, denoted by $\hat{A}$ and $\hat{H}$ respectively.

2. Find the estimates of the residuals $\hat{u}_t$ and $\hat{v}_t$ by replacing A with $\hat{A}$ in equation (4.1) and solving for u_t, denoting the solution by $\hat{u}_t$, $t = p + 1,..., T$. Similarly for $\hat{v}_t$, H is replaced by $\hat{H}$ in equation (2.5) and then this equation is solved for v_t, and the solution denoted by $\hat{v}_t$, $t = 1,..., T - p$. Note the least squares residuals need to be scaled. Following Thombs and Schucany (1990), the scaled

residuals are given as

$$\hat{U}_t = (\tilde{U}_t - \bar{U}) \cdot \sqrt{\frac{T-P}{T-P-KP-1}} \qquad \hat{U}_t = (\tilde{V}_t - \bar{V}) \cdot \sqrt{\frac{T-P}{T-P-KP-1}},$$

where $\bar{U}$ and $\bar{V}$ denote the element wise average of the vectors U_t, $t = p, ..., T$ and V_t, $t = 1, ..., T-p$, respectively.

3. Correct the coefficient matrices for bias as described in Section 3.2. using the bootstrap-after-bootstrap bias correction suggested by Kilian (1998), or in Section 3.1. for the bias correction based on the method proposed by Nicholls and Pope (1988). Following the notation given in Kim (2001), let $\hat{A}^c$ and $\hat{H}^c$ denote the bias corrected estimators for A and H, respectively.

4. Using the estimator $\hat{H}^c$, generate pseudo-data sets using the last p observed values of the time series $(y_1,..., y_T)$ as the starting values replacing, H with $\hat{H}^c$ in equation (2.5) as follows,

$$Y_t^* \quad = \quad \hat{\mu}^c + \hat{H}_1^c Y_{t+1}^* + \cdots + \hat{H}_p^c Y_{t+p}^* + \hat{V}_t^*,$$

where $\hat{V}_t^*$ are sampled with replacement from the least squares residuals found in (2). Find the least squares estimator based on $(Y_1^*,..., Y_T^*)$ for A and use the same bias correction as in (3). Denote the bias corrected estimator by $\tilde{A}^c$.

5. Find the bootstrap replicates of the AR forecast using,

$$Y_T^*(h) \quad = \quad \tilde{\nu}^c + \tilde{A}_1^c Y_T^*(h-1) + \cdots + \tilde{A}_p^c Y_T^*(h-p) + \hat{U}_{T+h}^*,$$

where $\hat{u}^*_{T+h}$ are sampled with replacement from the least squares residuals found in (2). In addition note that $Y^*_T(j) = y_{T+j}$ for $j \leq 0$.

6. Repeating steps (4) and (5), say B times, will result in the bootstrap forecast distribution $\{Y^*_T(h;\, i)\}_{i=1}^B$. This distribution can then be used to find the prediction intervals based on the percentile method or the percentile t-method.

Following Kim (1999), a bootstrap prediction interval for a future observation $Y_{k,Y+h}$ with at least $(1-\alpha)100\%$ coverage is given as,

$$BPI_{p,k} = \left[Y^*_{k,n}(h,\, \tau),\, Y^*_{k,n}(h,\, 1-\tau)\right],$$

where $\tau = 1/2(\alpha/K)$ and $Y^*_{k,n}(h,\, \tau)$ is the 100τ-th percentile of the k-th component of the bootstrap forecast distribution $\{Y^*_T(h;\, i)\}_{i=1}^B$. Taking the Cartesian product of all K intervals gives a joint prediction region with the desired coverage of at least $(1-\alpha)100\%$. The Bonferroni percentile t-intervals with a coverage rate of $(1-\alpha)100\%$ are given as the Cartesian product of,

$$BPI_{pt,k} = \left[\hat{y}^c_{k,n}(h) - z^*_{k,n}(h, 1-\tau)\sigma^c_k(h),\, \hat{y}^c_{k,n}(h) + z^*_{k,T}(h, 1-\tau)\sigma^c_k(h)\right],$$

where $\hat{y}^c_{k,n}(h)$ is the k-th component of the forecast using $\hat{A}^c$ in equation (4.2). Note that $z^*_{k,T}(h,\, \tau)$ is 100τ-th percentile of the standardized bootstrap distribution found as

$$z^*_{k,T}(h;\, i) \quad = \quad \frac{Y^*_{k,n}(h;\, i) - \hat{Y}^c_{k,n}(h)}{\hat{\sigma}^*_k(h)}.$$

Since the aforementioned prediction interval has the same form as prediction intervals found in the case of the normal distribution when the t-distribution is used, these intervals are called percentile-t- prediction intervals. Following Lütkepohl (1993), $\sigma_k^c(h)$ is the k-th diagonal element of the estimate of the matrix

$$\Sigma_{\hat{Y}}(h) = \Sigma_Y(h) + \frac{1}{T}\Omega(h), \tag{4.4}$$

where

$$\Sigma_Y(h) = \sum_{i=0}^{h-1} \Phi_i \Sigma_U \Phi_i', \text{ and} \tag{4.5}$$

$$\Omega(h) = \sum_{i=0}^{h-1}\sum_{j=0}^{h-1} tr\left[(\mathbf{B}')^{h-1-i}\Gamma^{-1}(\mathbf{B})^{h-1-j}\Gamma\right] \Phi_i \Sigma_U \Phi_j' \text{ and} \tag{4.6}$$

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \nu & A_1 & \dots & A_p \\ 0 & & I_{K(p-1)} & 0 \end{bmatrix}, \tag{4.7}$$

and $\Gamma = \text{plim}(ZZ')/T$ where Z is defined in equation (2.7). Equation (4.4) holds if Y_t is a stationary time series.

In practice, it is not possible to obtain $\Omega(h)$ for all h without knowledge of the matrices $\mathbf{B}$, Σ_U and Γ. Therefore, the use of consistent estimators for these matrices is needed. For the matrix $\mathbf{B}$, replacing ν, A_1,..., A_p with their least squares estimators given in Theorem 2.14, Σ_U is replaced by $\hat{\Sigma}_U$, which is given in equation (3.3). $\hat{\Phi}$ is found using the least squares estimators $\hat{\nu}$, $\hat{A}_1$,..., $\hat{A}_p$. These estimators are described in more detail in Sections 2.3. and 2.4., and a natural estimator for Γ is

$\hat{\Gamma} = ZZ'/T$, with Z defined in equation (2.7). Following the notation of Lütkepohl (1993), the resulting estimator is then denoted by $\hat{\Sigma}_{\hat{Y}}(h)$.

4.3.2. Bootstrap Prediction Intervals Based on the Forward Representation of the Time Series. The following procedure for bootstrap prediction intervals is based on a method proposed by Pascual, Romo, and Ruiz (2004) for the univariate case. This method does not require the backward representation of the time series and, as suggested by Pascual, Romo, and Ruiz (2004), can be used to find prediction intervals for MA, ARMA or ARIMA. In addition, Kim (2001) mentions that if the innovations are not normally distributed, the assumption of independence for the backward innovations fails. However, in a simulation study done by Kim (2001) and (2004), using non-normal distributions for the innovations did not negatively effect the coverage probabilities of the bootstrap prediction intervals. Following the suggestions of Pascual, Romo, and Ruiz (2004) and including the bias correction suggested by Kim (2001), the following method is proposed to find prediction intervals.

1. Given $(y_1,..., y_T)$, estimate the coefficient matrices $A = (\nu, A_1,..., A_p)$ using least squares estimation, denoted by $\hat{A}$.

2. Find the estimates of the residuals, $\hat{U}_t$, by replacing A by $\hat{A}$ in equation (4.1) and solving for $\tilde{U}_t$, $t = p + 1,..., T$. Note the least squares residuals need to be scaled. Following Thombs and Schucany (1990), the scaled residuals are given

as

$$\hat{U}_t = (\tilde{U}_t - \bar{U}) \cdot \sqrt{\frac{T-P}{T-P-KP-1}},$$

where $\bar{U}$ denotes the element wise average of the vector U_t.

3. Correct the coefficient matrices for bias as described in Section 3.2. for the bootstrap-after-bootstrap bias correction, or in Section 3.1. for the bias correction based on the method proposed by Nicholls and Pope. Following the notation given in Kim (2001), let $\hat{A}^c$ denote the bias corrected estimators for A.

4. Using the estimator $\hat{A}^c$ generate pseudo data sets using the first p observed values of the time series $(y_1,..., y_T)$ as the starting values replacing A by $\hat{A}^c$ in equation (4.1) as,

$$Y_t^* = \hat{\nu}^c + \hat{A}_1^c Y_{t-1} + \cdots + \hat{A}_p^c Y_{t-p} + \hat{U}_t^*,$$

where $\hat{U}_t^*$ are sampled with replacement from the least squares residuals found in (2). Find the least squares estimator based on $(Y_1^*,..., Y_T^*)$ for A and use the same bias correction as in (3). Denote the bias corrected estimator based on forward resampled time series by $\tilde{A}^{cf}$.

5. Find the bootstrap replicates of the AR forecast using,

$$Y_T^*(h) = \tilde{\nu}^{cf} + \tilde{A}_1^{cf} Y_T^*(h-1) + \cdots + \tilde{A}_p^{cf} Y_T^*(h-p) + \hat{U}_{T+h}^*,$$

where $\hat{U}^*_{T+h}$ are sampled with replacement from the least squares residuals found in (2). In addition, note that $Y_T(j)^* = y_{T+j}$ for $j \leq 0$.

6. Repeating steps (4) and (5), say B times will result in the bootstrap forecast distribution $\{Y^*_T(h;\, i)\}_{i=1}^{B}$. This distribution can then be used to find the prediction intervals based on the percentile method or the percentile t-method.

Note the actual calculations of the prediction intervals are the same for both the backward bootstrap prediction intervals and the forward bootstrap prediction intervals. Kim (2001) gives a proof of the asymptotic properties of the backward bootstrap prediction intervals. Since both bias adjustments converge to zero in probability as $T \to \infty$, this adjustment does not change the proof of the asymptotic properties of the backward bootstrap intervals. However, the proof of Kim (2001) is modified to show the asymptotic properties of the forward bootstrap intervals. These proofs only consider VAR(p) time series. Pascual, Romo, and Ruiz (2004) give a proof for univariate ARMA(p, q) and ARIMA(p, d, q), where d is known. The extension of this proof to the multivariate ARMA(p, q) case is of future research interest. Unlike Pascual, Romo, and Ruiz (2004) state in the proof for the univariate case, the multivariate case is, however, not a direct application of the results given by Freedman (1984). It will still be necessary to show that the bootstrapped estimators of the VARMA model converge to the true parameter values.

5. ASYMPTOTIC RESULTS

5.1. ASYMPTOTIC RESULTS FOR BOOTSTRAPPED LEAST SQUARES ESTIMATORS

Freedman (1984) established results for properties of the least squares estimators in stationary linear models for the bootstrapped time series. In the following, the results are modified for the time series model given in equations (2.1) and (2.4) that are the focus of this research. The results herein are based on a model that is equivalent to the model given in equation (2.4). Freedman (1984) defines the following model,

$$Y_t = Y_t A + Y_{t-1} B + X_t C + U_t. \tag{5.1}$$

In the equation (5.1), A, B and C are unknown coefficient matrices that need to be estimated, U_t is the vector of endogenous (correlated with the disturbances) variables at time t, and X_t is a vector of exogenous (uncorrelated with the disturbances) variables at time t. For the model given in equation (2.4) it is easily seen that $A = C = 0$. Freedman suggests using two-stage least squares to estimate the unknown parameters and then utilizing the estimators to compute the vector of disturbances, U_t, at time, t, by,

$$\hat{U}_t = Y_t - Y_t \hat{A} - Y_{t-1} \hat{B} - X_t \hat{C}. \tag{5.2}$$

Denoting the pseudo-data with as asterisk, $Y_1^*,\dots,Y_T^*$ will be constructed following the iteration

$$Y_t^* \quad = \quad (Y_{t-1}^*\hat{B} + X_t\hat{C} + \hat{U}_t^*)(I - \hat{A})^{-1}, \tag{5.3}$$

where the $\hat{U}_t^*$ are randomly drawn using the empirical distribution from $\hat{U}_1,\dots,\hat{U}_T$ and Y_0^* is set to equal Y_0. Note that the above rule applies for $t = 1,\dots,T$. Freedman (1984), having the exogenous variable X_t, needs to define a variable V_t. In the consideration here, it is the case where $X_t = 0$ for all $t \in \mathbb{Z}$, $V_t = Y_{t-1}$, which will be used instead of V_t. Following the notation given by Freedman (1984), omitting quantities that are not required for the prediction intervals considered here, define,

$$S = E[Y_{t-1}'Y_{t-1}], \qquad S_T^* = 1/T\sum_{t=1}^{T} Y_{t-1}^{*\prime}Y_{t-1}^*, \text{ and} \tag{5.4}$$

$$\Delta_T = 1/T\sum_{t=1}^{T} Y_{t-1}'U_t, \qquad \Delta_T^* = 1/T\sum_{t=1}^{T} Y_{t-1}^{*\prime}\hat{U}_t^* \tag{5.5}$$

where Y_{t-1}^* and $\hat{U}_t^*$ denote the bootstrapped version Y_{t-1} and U_t, respectively. Denote the distribution functions of $\sqrt{T}\Delta_T$ and $\sqrt{T}\Delta_T^*$, conditional on the data, by $F_{\sqrt{T}\Delta_T}(\cdot\,|\,Y_1,\dots,Y_T)$ and $F_{\sqrt{T}\Delta_T^*}(\cdot\,|\,Y_1,\dots,Y_T)$, respectively. In addition, denote the limiting distribution of $\sqrt{T}\Delta_T$ by $F_\Delta(\cdot\,|\,Y_1,\dots,Y_T)$. Let Δ denote the limit of Δ_T and denote the distribution of Δ by $F_\Delta(\cdot)$. Having defined the quantities in equation (5.4) and (5.5), Freedman states Theorem 5.1.

Theorem 5.1

Let $y_1, \ldots, y_T$ be an observed series satisfying the difference equation given in equation (5.1). Using the empirical distribution, resample from $\hat{U}_1, \ldots, \hat{U}_T$ given in equation (5.2) and utilize equation (5.3) to calculate $Y_1^*, \ldots, Y_T^*$. Then, among almost all sequences $Y_1^*, \ldots, Y_T^*$,

(a) $\lim_{T\to\infty} P\Big(|S_T^* - S| \leq \varepsilon \,\Big|\, Y_1, \ldots, Y_T\Big) = 1$, and

(b) $\lim_{T\to\infty} F_{\sqrt{T}\Delta_T^*}(\cdot \,|\, Y_1, \ldots, Y_T) \to F_\Delta(\cdot)$.

Proof: The proof of this theorem can be found in Freedman (1984).

Part (a) of Theorem (5.1) states that S_T^* converges to S in conditional probability and part (b) states that the conditional law of $\sqrt{T}\Delta_T^*$ has the same limit as the unconditional law of $\sqrt{T}\Delta_T$. This theorem is necessary to show that the bootstrapped estimators of Π, denoted by Π^*, and A^*, the bootstrapped estimator of A, converge in probability to the true parameter matrix. The next corollary establishes this result.

Corollary 5.2

Let $y_1^*, \ldots, y_T^*$ be a bootstrapped time series generated using equation (5.3). Then the least squares estimator of Π based on the bootstrapped sample converges in probability to Π and $\hat{A}^*$ converges in probability to A.

Proof: The least squares estimators of the bootstrapped model modified from the model given in equation (2.4) are given in equation (2.9) by replacing $\mathbf{W}_T$ by, $\mathbf{W}_T^*$, as

$$\hat{\Pi}^* = \Pi + w_T^* S_T^{*-1},$$

where $w_T^* = 1/T \sum U_t^* \mathbf{W}_{t-1}^{*'}$ and $S_T^* = 1/T \sum \mathbf{W}_{t-1}^* \mathbf{W}_{t-1}^{*'}$. From Theorem 5.1 it follows that $S_T^* \xrightarrow{p} S = E[Y_{t-1}' Y_{t-1}] < \infty$ and that w_T^* converges in distribution to the same limit as w_T. Note that Lütkepohl (1993) states plim$UZ'/T = 0$, which implies that plim$w_T = 0$ and hence plim$w_T^* = 0$. From this it follows from Theorem 2.11 that, $\hat{\Pi}^*$ converges in probability to Π, which implies that $\hat{A}^*$ converges in probability to A.

In addition, one more lemma given by Freedman is necessary. This lemma establishes that $\hat{U}_t^*$ converges weakly to U_t. The lemma states a strong result, which yields Theorem 5.9.

Lemma 5.3

Let $\hat{U}_1$,..., $\hat{U}_T$ be found using equation (5.2). Under the assumption that the innovations, U_t, are i.i.d., then the empirical distribution function of $\hat{U}_t^*$ converges to the true distribution of U_t.

Proof: The proof can be found in Freedman (1984).

The aforementioned definitions, theorems, propositions, corollaries and lemmas constitute all the necessary results that help in proving the consistency of the proposed bootstrap method.

5.2. RESULTS FOR BOOTSTRAP METHODS

The main question remaining is: "When does the bootstrap method work for bootstrap prediction bound for multivariate time series?" To be able to find a prediction interval for a future value it is necessary to find an estimate of the distribution of that future value. In the parametric case, it is sometimes possible to find a pivotal quantity, whose distribution does not depend on the unknown parameters, which is then used to find lower and upper prediction bounds. It is not always possible to find such pivotal quantities. In this case, a normal approximation may be used, which, if the sample size is small, may not always be accurate. The method proposed in this dissertation for prediction intervals for AR(p) time series is based on finding a bootstrap estimate of the distribution.

Definition 5.4

Let $y_1,...y_T$ and $y_1^*,...y_T^*$ be time series with innovations U_t and $\hat{U}_t$, with distribution function F_U and the empirical distribution function $\hat{F}_U$, respectively. Then the quantity, about which statistical inferences are desired, is defined by,

$$Q(Y_1, ..., Y_T;\ F_U) \quad \text{and respectively} \quad Q(Y_1^*, ..., Y_T^*;\ \hat{F}_U).$$

When considering prediction intervals h steps ahead, $Q(Y_1, ..., Y_T;\ F_U)$ is Y_{T+h}, whereas $Q(Y_1^*, ..., Y_T^*;\ \hat{F}_U)$ is the bootstrapped h step ahead value $Y_T^*(h)$. The sampling distribution of $Q(Y_1, ..., Y_T;\ F_U)$ is defined as

$$G_{F,n}(q) \;=\; P\Big(Q(Y_1, ..., Y_T;\ F) \leq q \,|\, F_U\Big),$$

and the bootstrap distribution of $Y_T^*(h)$ is defined as

$$G_{\hat{F},T}(q) \;=\; P\Big(Q(Y_1^*,...,Y_T^*;\,\hat{F}) \leq q \,|\, \hat{F}\Big). \tag{5.6}$$

Resampling with replacement from $\hat{F}_U$ generates the bootstrap distribution of $Y_T^*(h)$. Following the procedure described in Section 4.3., the bootstrapped time series is used to estimate the unknown parameter A. This estimate and the last p observations of the original observed time series are used to find the future value $Y_T^*(h)$. Consider the AR(1) case with $T = 50$ and a one step ahead prediction. Then,

$$Y_T^*(1) \;=\; \nu^* + \tilde{A}_1^* Y_T + \hat{U}_{T+1}^*,$$

where $\hat{U}_{T+1}^*$ is randomly selected from the empirical distribution of the observed U_ts and ν^* and A_1^* are the bootstrapped estimators of ν and A_1. Then, since ν^* and A_1^* are based on the bootstrapped time series, $Y_1^*,...,Y_T^*$, there are $49^{49} \approx 6 \times 10^{82}$ different possible values for ν and A_1. Note that this is assuming that all the $\hat{u}_t$s are distinct. The probability of two $\hat{u}_t$'s being equal is zero, since it is assumed that they are originally drawn from a continuous distribution. Each of these possible choices of ν and A_1 can be combined with the random draw from the 49 $\hat{u}_t$ resulting in a total of approximately 3×10^{84} possible values of $Y_T^*(1)$. So the bootstrap distribution of Y_T^* is given by assigning equal probability to any one of these possible values.

The following theorem shows under what assumptions the bootstrap prediction intervals are consistent, i.e., the bootstrap prediction bounds converge to the true prediction bounds. This result was not found in any previous papers concerning bootstrap prediction intervals; Thombs and Schucany (1990), Kim (2001) and Pascual, Romo, and Ruiz (2004) assumed that this result is known. A proof of the result showing that the weak convergence suffices for consistency of the simultaneous bootstrap prediction intervals, is given here. Bonferroni's method is used to find simultaneous prediction intervals for VAR(p) time series. The bounds of the intervals are found separately for each component of the vector Y_{T+h}. To denote this, the subscript k is added, i.e. $G_{k,F,\infty}$ denotes the marginal distribution function of $Y_{k,T+h}$.

Define $q_{k,\alpha}$ and $q_{k,\alpha}(T)$ by

$$q_{k,\alpha} = \inf\{q : G_{k,F,\infty}(q) = 1-\alpha\}, \text{ and} \tag{5.7}$$

$$q_{k,\alpha}(T) = \inf\{q : 1-\alpha \le G_{k,F,T}(q) < 1-\alpha+1/T\}, \tag{5.8}$$

Then the following theorem shows that $q_{k,\alpha}(T)$ converges to $q_{k,\alpha}(T)$ as $T \to \infty$.

Theorem 5.5 Consistency of the Bootstrap

Let $y_1, \ldots, y_T$ be an observed series satisfying the difference equation (2.1) and assume the CDF of $Y_{k,T+h}$, $F_{Y_{k,T+h}} = G_{k,F_U,\infty}(q_k)$ is increasing in a neighborhood of $q_{k,\alpha}$. Let $q_{k,\alpha}$ and $q_{k,\alpha}(T)$ be as defined in equations (5.7) and (5.8). If the bootstrap distribution $G_{\hat{F},T}(q)$ converges to $G_{F,\infty}(q)$, then the bootstrap prediction bounds are consistent:

$$P(Y_{k,T+h} \le q_{k,\alpha}(T)) \to 1-\alpha \qquad \text{as } T \to \infty.$$

Proof: Note that $q_{k,\alpha}(T)$ is used as the upper prediction bound giving coverage of at least $1-\alpha$, and $q_{k,\alpha}$ is the true upper prediction bound giving exact coverage of $1-\alpha$. It is possible to find $\Delta_T < 1/T$ such that $G_{\hat{F},T}(q_{k,\alpha}(T)) = 1 - \alpha + \Delta_T$. In addition $\Delta_T < \varepsilon/2$ infinitely often (i.o). It is necessary to show that $q_{k,\alpha}(T)$ converges to $q_{k,\alpha}$. Assume the opposite, thus there exists a $\delta > 0$ such that $|q_{k,\alpha}(T) - q_{k,\alpha}| > \delta$ infinitely often (i.o.). Since $G_{k,F,\infty}(q)$ is a continuous monotone increasing function, there exists $\varepsilon > 0$ such that,

$$|G_{k,F,\infty}(q_{k,\alpha}(n)) - G_{k,F,\infty}(q_{k,\alpha})| > \varepsilon \qquad \text{i.o.}$$

Note that under the assumption that $G_{F,T}(q) \to G_{k,F,\infty}(q)$ as $T \to \infty$ it follows

$$|G_{k,F,\infty}(q_{k,\alpha}(T)) - G_{\hat{F},T}(q_{k,\alpha}(T))| < \varepsilon/2 \qquad \text{as } T \to \infty.$$

Now

$$\begin{aligned}
& |G_{k,F,\infty}(q_{k,\alpha}(T)) - G_{k,F,\infty}(q_{k,\alpha})| > \varepsilon \qquad \text{i.o.} \\
\Leftrightarrow\ & |G_{k,F,\infty}(q_{k,\alpha}(T)) - G_{k,F,\infty}(q_{k,\alpha}(T)) + G_{k,F,\infty}(q_{k,\alpha}(T)) - G_{k,F,\infty}(q_{k,\alpha})| > \varepsilon \qquad \text{i.o.} \\
\Leftrightarrow\ & |G_{k,F,\infty}(q_{k,\alpha}(T)) - G_{k,F,\infty}(q_{k,\alpha}(T)) + (1 - \alpha + \Delta_T) - (1 - \alpha)| > \varepsilon \qquad \text{i.o.} \\
\Leftrightarrow\ & |G_{k,F,\infty}(q_{k,\alpha}(T)) - G_{k,F,\infty}(q_{k,\alpha}(T))| + \Delta_T > \varepsilon \qquad \text{i.o.}
\end{aligned}$$

But

$$|G_{k,F,\infty}(q_{k,\alpha}(T)) - G_{k,F,\infty}(q_{k,\alpha}(T))| + \Delta_T < \varepsilon/2 + \varepsilon/2 = \varepsilon \qquad \text{i.o.},$$

hence a contradiction. Thus $q_{k,\alpha}(T) \to q_{k,\alpha}$.

Hence the bounds of the bootstrap prediction intervals converge to the true bounds. Note that the result is only shown for the upper prediction bound. It can be easily established for the lower bound as well, thus it is omitted here. Thus, it is not necessary to have uniform convergence of the bootstrap distribution function. Thombs and Schucany (1990) and Kim (2001) actually show the weak convergence in their proofs, which, with the above result, suffice for the consistency of the bootstrap method.

In some cases it is not feasible to obtain the exact bootstrap distribution function. The bootstrap distribution function is given as $G_{k,F,\infty}(q)$, as defined in equation (5.6). In case of the bootstrap distribution, $G(Y_1^*, ..., Y_T^*;\ \hat{F})$ of $Y_T^*(h)$ there are too many possible values, even when $T = 50$, to be able to calculate the correct bootstrap distribution. Since the exact bootstrap distribution function is not available, a Monte Carlo approximation is used in the simulation study. Since a random sample with replacement of size B is taken from the distribution, $G(Y_1^*, ..., Y_T^*;\ \hat{F})$, the empirical distribution of that sample converges to the true distribution, $G(Y_1^*, ..., Y_T^*;\ \hat{F})$, as $B \to \infty$ by Theorem 2.13. Thus, this approximation used in the simulation study is asymptotically valid.

5.3. ASYMPTOTIC VALIDITY OF PREDICTION INTERVALS BASED ON THE FORWARD REPRESENTATION OF THE TIME SERIES

This section will cover the asymptotic results that have been established by Kim (1999), (2001) and (2004), and Pascual, Romo, and Ruiz (2004) and extend them to the case of the forward bootstrap prediction intervals for VAR(p) time series. Kim (2001) states and proves the following theorem:

Theorem 5.6 Asymptotic Validity of the Bootstrap using the Backward Representation

Let $y_1,..., y_T$ be an observed series satisfying the difference equation (2.1). Under the assumption that the U_t's are i.i.d. normal distributed, and utilizing the procedure given in Section 4.3.1., conditionally on the data $y_1,..., y_p$, then

(a) $\lim_{T\to\infty} P(|\tilde{A}^c - A| \geq \varepsilon \,|\, Y_1,...T_T) = 0$ and $\lim_{T\to\infty} P(|\hat{A}^c - A| \geq \varepsilon \,|\, y_1,...y_T) = 0$,

(b) $\lim_{T\to\infty} P(|\hat{H}^c - H| \geq \varepsilon \,|\, Y_1,...T_T) = 0$, and

(c) $\lim_{T\to\infty} P(Y_n^*(h) \leq y \,|\, Y_1,..., Y_T) = P(Y_{n+h} \leq y \,|\, y_1,..., y_T)$.

The proof of this theorem can be found in Kim (2001).

Kim (2001) also notes that the theorem is only proved for stationary AR processes, but he investigates the properties of the bootstrap prediction intervals for time series with unit roots using a simulation study. In addition, Kim (2001) notes that the independence of the innovations V_t in equation (2.5) rely on the normality assumption of the innovations, U_t, in equation (2.1). In the simulation studies conducted by Kim (2001) and (2004), he showed that the use of non-normal innovations does not affect the coverage negatively.

Pascual, Romo, and Ruiz (2004) state the following theorem, which establishes the asymptotic validity of the bootstrap prediction intervals, for the univariate case, using the forward representation.

Theorem 5.7

Let $y_T = (y_{T-n+1},..., y_T)$ be a realization of an univariate ARIMA(p, d, q) process $\{Y_T\}$ with $\mathbb{E}(U_t^4) < \infty$ and let the roots of the autoregressive and moving average polynomials satisfy the usual stationary and invertability conditions, respectively. Let $(\hat{A},\ \hat{M})$ be any M-estimate of $(A,\ M)$ and let Y^*_{T+h} be obtained following steps 1 to 5 given in Pascual, Romo, and Ruiz (2004). Then, given $y_1,..., y_T$, Y^*_{T+h} converges weakly in probability to Y_{T+h} as T tends to infinity.

The proof of this theorem can be found in Pascual, Romo, and Ruiz (2004).

Theorem 5.6 is modified to match the forward representation of a VAR(p) process. Extending Theorem 5.7 to the VARMA(p, q) time series is of future research interest, since the convergence of the bootstrapped estimators of the VARMA$(p,\ q)$ time series to the true parameters still needs to be established. The theorem for the forward representation is the equivalent result of the theorem given by Kim (2001).

Theorem 5.8 Asymptotic Validity of the Prediction Intervals using the Forward Representation of the Time Series

Let $y_1,..., y_T$ be a realization of a time series following the difference equation (2.1) with U_t being a standard white noise process with mean zero and variance covariance matrix Σ_U. Then, following the procedure described in Section 4.3.2.:

(a) $\lim_{T\to\infty} P(|\tilde{A}^{cf} - A| \geq \varepsilon \,|\, Y_1,..., Y_T) = 0$ and $\lim_{T\to\infty} P(|\hat{A}^{cf} - A| \geq \varepsilon \,|\, Y_1,..., Y_T) = 0$, for all $\varepsilon > 0$, and

(b) $\lim_{T\to\infty} P(Y_n^*(h) \leq y \,|\, Y_1,..., Y_T) = P(Y_{n+h} \leq y \,|\, Y_1,..., Y_T)$.

Proof: Writing the time series given in (2.1) as

$$\mathbf{W}_t = \Pi \mathbf{W}_{t-1} + \mathbf{U}_t,$$

where $W_t = (Y_t',..., Y_{t-p+1}')'$ and $\mathbf{U}_t = (U_t', 0,..., 0)'$ are $Kp \times 1$ vectors and the $Kp \times Kp$ matrix Π is given as

$$\Pi = \begin{bmatrix} A_1 & \dots & A_p \\ & I_{Kp-K} & \mathbf{0} \end{bmatrix},$$

and the least squares bootstrapped estimator of Π is given as

$$\hat{\Pi}^* = \left(\sum \mathbf{W}_t^* \mathbf{W}_{t-1}^{*\prime}\right)\left(\sum \mathbf{W}_{t-1}^* \mathbf{W}_{t-1}^{*\prime}\right)^{-1} = \Pi + \mathbf{w}_T^* S_T^{*-1},$$

where $\mathbf{w}_T^* = 1/T \sum \mathbf{U}_t^* \mathbf{W}_t^{*\prime}$ and $S_T^* = 1/T \sum \mathbf{W}_{T-1}^* \mathbf{W}_{T-1}^{*\prime}$. From Theorem 5.1 and the Corollary 5.2, it follows that $\hat{\Pi}^*$ is a consistent estimator of Π, hence $\hat{A}^*$

converges in probability to A. Both bias corrections, denoted as $\hat{\Psi}$, as $T \to \infty$ go to zero in probability, thus $\tilde{A}^{cf} = \hat{A}^{*} - \hat{\Psi}$ converges in probability to A, which follows from results of convergence in probability provided in Section 2.2.

Since the $\hat{U}_t$ are based on the least squares estimator $\hat{A}$, which converges a.s. to A, Lemma 5.3 is applicable. Hence, $\hat{U}_t^*$ converges weakly to U_t. Since convergence in probability implies convergence in distribution, $\tilde{A}^c \xrightarrow{d} A$ and $Y_T(h) = \tilde{A}_1^{cf} Y_T(h-1) + ... + \tilde{A}_p^{cf} Y_T(h-p) + U_{T+h}^*$ converges in distribution to Y_{T+h} by Slutsky's Theorem.

The proof of Theorem 5.8 shows that $\tilde{A}^{cf}$ converges to A in probability and that $Y_T^*(h)$ converges weakly to Y_{T+h}. Both Kim (2001) and Thombs and Schucany (1990) show only the weak convergence. Pascual, Romo, and Ruiz (2004) show in the proof for the univariate forward representation that $Y_T^*(h)$ converges weakly in probability to Y_{T+h}. Davison and Hinkley (2003) state that this result is sufficient for the bootstrap to be consistent. As seen in Section 5.2. this result is not necessary for simultaneous bootstrap prediction regions to be consistent. The following proof can be extended to the case of weak convergence in probability if Slutsky's Theorem 2.10 holds for weak convergence in probability.

Theorem 5.9 Convergence of the Bootstrap Distribution

Let $y_1, ..., y_T$ be a realization of a time series following the difference equation (2.1) with U_t being a standard white noise process with mean zero and variance covariance matrix Σ_U. Then, following the procedure described in Section 4.3.2., $Y_T^*(h)$ converges weakly to Y_{T+h}.

Proof: Bootstrapping the time series following the difference equation given in (2.1) as

$$\mathbf{W}_t^* = \hat{\Pi}\mathbf{W}_{t-1}^* + \mathbf{U}_t^*,$$

where $W_t^* = (Y_t^{*'},..., Y_{t-p+1}^{*'})'$ and $\mathbf{U}_t = (U_t^{*'}, 0,..., 0)'$ are $Kp \times 1$ vectors and the least squares estimator of the $Kp \times Kp$ matrix Π, $\hat{\Pi}$ is given as

$$\hat{\Pi} = \left(\sum \mathbf{W}_t \mathbf{W}_{t-1}'\right)\left(\sum \mathbf{W}_{t-1}\mathbf{W}_{t-1}'\right)^{-1} = \Pi + \mathbf{w}_T S_T^{-1},$$

computed using the observed time series and then correcting for bias. The bootstrap least squares estimator of Π using the bootstrapped time series is given as

$$\hat{\Pi}^* = \left(\sum \mathbf{W}_t^* \mathbf{W}_{t-1}^{*}{}'\right)\left(\sum \mathbf{W}_{t-1}^*\mathbf{W}_{t-1}^{*}{}'\right)^{-1} = \Pi + \mathbf{w}_T^* S_T^{*-1}.$$

From Theorem 5.1 and the Corollary 5.2, it follows that $\hat{\Pi}^*$ is a consistent estimator of Π, hence $\hat{A}^*$ converges in probability to A. In addition, Lemma 5.3 establishes that U_{t+h}^* converges weakly a.s. to U_{t+h} for $h \in \mathbb{N}$.

Note that by applying the difference equation to the time series, it is possible to represent $Y_T^*(h)$ as a function of $y_{T-p+1},...y_T$, i.e.,

$$\begin{aligned} Y_T^*(h) = \; & g_0(\tilde{A}^{cf}) + g_1(\tilde{A}^{cf})Y_T + \ldots + g_p(\tilde{A}^{cf})Y_{T-p+1} + \\ & h_1(\tilde{A}^{cf})\hat{U}_{T+1}^* + \ldots + h_{p-1}(\tilde{A}^{cf})\hat{U}_{T+p-1}^* + \hat{U}_{T+h}^*. \end{aligned}$$

Note that the functions g_i and h_i are polynomials in $\tilde{A}^{cf}$ and thus continuous functions. Thus, $g_i(\tilde{A}^{cf})$ converges in probability to $g_i(A)$ and, respectively, $h_i(\tilde{A}^{cf})$ converges in probability to $h_i(A)$ by results for stochastic convergence (Theorem 2.11). It follows from Slutsky's Theorem 2.10, that $h_i(\tilde{A}^{cf})\hat{U}^*_{T+i}$ converges weakly to $h_i(A)U_{T+i}$.

Since convergence in probability implies weak convergence, it follows again from Slutsky's Theorem that $Y^*_T(h)$ converges weakly to Y_{T+h}.

This completes the proof of the asymptotic validity of the bootstrap method for forward prediction. However, asymptotic validity is not sufficient to ensure the usefulness of the proposed method. In the following two sections, a simulation study is done to show that the proposed method has good small sample properties.

6. THE SIMULATION

To verify that the proposed method gives good small sample results, a simulation program was written in FORTRAN, which is given in the appendix C.. The compiler used is Intel 8.0 compiler suite. Following the recommendation of IMSL the seed was set to equal 123457. The code was run on a dual P4-Xeon® 2.0Ghz with 4 GB of RAM. The simulation results for the backward representation found in the simulation study considered for this dissertation were compared to the results of the simulation programs provided by Kim (2001) and (2004). In addition, each program was verified by running the program and comparing the results of each step with results obtained from calculation done utilizing Maple. During the process of verifying the FORTRAN code, a difference in the implementation of the bootstrap-after-bootstrap method employed by Kim (2001) was found. To estimate the bias, the average of B bootstrapped estimates of the parameters is calculated and then used to calculate the bias, $\hat{\Psi}$. Then, to save computing time, this bias estimate is used when calculating the estimates for predicting future values. Instead of estimating the bias and using this estimated bias to correct the bootstrapped estimators for the bias, Kim reestimated the bias each time using the average of the B bootstrapped estimates. Following the notation from Section 3.2., let $\bar{\hat{A}}^*$ denote the B bootstrapped estimators for the bias correction and $\hat{A}^*$ the bootstrapped estimator for the prediction intervals. Hence, Kim's bias corrected estimate is given by $\hat{A}^{*c} = 2\hat{A}^* - \bar{\hat{A}}^*$, whereas the method suggested by Kilian

(1998) says to use $\hat{A}^{*c} = \hat{A}^{*} - \hat{\Psi}$. In the simulation carried out by Kim (2001), his proposed method gave very conservative results for some parameter combinations. Utilizing the method proposed by Kilian (1998), the coverage probabilities for the method utilizing the backward representation of the time series are closer to nominal coverage.

The parameter combinations for VAR(1) and VAR(2) are chosen to be the same as those used by Kim in his papers (1999), (2001) and (2004). For the VAR(1) model, Kim ran the simulation with the parameter combinations given in Table 6.1. Even though the mathematical derivations assume stationary models, it is of interest to see whether the prediction intervals are applicable when the true model is a unit root process. In addition, it is examined how good the coverage of the prediction intervals are, if the time series has roots close to unity. Similarly for the VAR(2) case, several parameter combinations with and without unit roots are examined. The parameter combinations used in this case are given in Table 6.2. It is also of interest to see that the average coverage probabilities of the intervals hold if the order of the process is not known. Assuming the order is not known, Akaike's Information Criteria (AIC) is used to determine the order, p of the time series. Details for AIC can be found in Lütkepohl (1993). This p is then used to bootstrap the time series and calculate the prediction regions. Clearly, it is expected that order selection for a stationary VAR(1) works well, which is seen when running the simulation. However, if the underlying time series is VARMA$(1,1)$, it is of interest to see how close the average coverage is to the true coverage. In this case, the VAR representation will have infinite order. The parameter combinations for the VARMA$(1,1)$ case are given in Table 6.3.

Kim (1999) suggests using $\text{vech}\Sigma_U = (1,.5,1)'$ as the covariance of the innovations, where vech is the column stacking operator that stacks elements on and below the diagonal. In his (2001) paper he uses $\text{vech}\Sigma_U = (1,.3,1)'$, which is also used in the simulation studies considered in this dissertation. In addition, some of the simulations are run with an inflated variance of $\text{vech}\Sigma_U = 2 \cdot (1,.3,1)$.

The backward representation is only valid if the innovations are normally distributed, since this assumption is required for the independence of the innovations. In this case the innovations are uncorrelated but not independent. Since the bootstrap requires the resampling of independent observations, it is of interest to see whether the backward representation works, at least in an approximate way, for non-Gaussian innovations. Kim (1999) notes that there is the possibility of using the relationship between the forward and backward innovations to resample non-Gaussian innovations. Kim (1999), (2001) and (2004) did not use this technique for the non-Gaussian distributions, but did consider several non-Gaussian distributions for the innovations. He suggests using a bivariate χ^2-distribution with 4 degrees of freedom, and a bivariate t-distribution with 5 degrees of freedom. These random variables are scaled so that they have the covariance of $\text{vech}\Sigma_U = (1,.3,1)'$ and zero mean. The χ^2-distribution represents an asymmetric distribution and the t-distribution has heavy tails. In addition, in this dissertation the exponential distribution with $\lambda = 1$ is considered, which is scaled to have the covariance of $\text{vech}\Sigma_U = (1,.3,1)'$ and centered to have mean zero. This is achieved by generating

Table 6.1. Parameter Combinations for VAR(1)

	Model 1	Model 2	Model 3	Model 4	Model 5
$A_1 =$	$\begin{bmatrix} -.5 & 0 \\ .5 & .5 \end{bmatrix}$	$\begin{bmatrix} .5 & 0 \\ .5 & .5 \end{bmatrix}$	$\begin{bmatrix} .97 & 0 \\ .5 & .5 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ .5 & .5 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Roots	$2, 2$	$2, 2$	$2, 1.0309$	$2, 1$	$1, 1$

Table 6.2. Parameter Combinations for VAR(2)

	Model 6	Model 7	Model 8	Model 9
$A_1 =$	$\begin{bmatrix} .9 & 0 \\ .5 & -.7 \end{bmatrix}$	$\begin{bmatrix} .9 & 0 \\ .5 & -.5 \end{bmatrix}$	$\begin{bmatrix} .2 & 0 \\ .5 & -.5 \end{bmatrix}$	$\begin{bmatrix} .4 & 0 \\ .5 & 1.4 \end{bmatrix}$
$A_2 =$	$\begin{bmatrix} -.2 & 0 \\ -.8 & -.1 \end{bmatrix}$	$\begin{bmatrix} -.2 & 0 \\ -.8 & -.125 \end{bmatrix}$	$\begin{bmatrix} -.5 & 0 \\ -.8 & -.125 \end{bmatrix}$	$\begin{bmatrix} .45 & 0 \\ -.8 & -.45 \end{bmatrix}$
Roots	$-5, -2, 2, 2.5$	$2, 2.5, 2 \pm 2i$	$.2 \pm 1.4i, -2 \pm 2i$	$-2, -1.11, 1.11, 2$

two independent random variables with the appropriate underlying distribution, X_1 and X_2, with mean μ and variance σ^2 and then defining Y_1 and Y_2 as

$$Y_1 = \frac{(X_1 - \mu)}{\sigma} \qquad Y_2 = \rho\frac{(X_1 - \mu)}{\sigma} + \sqrt{1-\rho^2}\frac{(X_2 - \mu)}{\sigma},$$

where ρ is the desired correlation between Y_1 and Y_2. In the simulation study considered here, $\rho = .3$ and $.6$.

To generate the time series, the first 300 generated values were omitted to ensure that the generated time series is stationary. Kim (2001) and (2004) calculated prediction regions for time series of length $T = 50$, 100 and 200. The simulations

Table 6.3. Parameter Combinations for VARMA(1, 1)

	Model 11	Model 12	Model 13	Model 14	Model 15
$A_1 =$	$\begin{bmatrix} .5 & 0 \\ .5 & .5 \end{bmatrix}$	$\begin{bmatrix} .5 & 0 \\ .5 & .5 \end{bmatrix}$	$\begin{bmatrix} .9 & 0 \\ .5 & .9 \end{bmatrix}$	$\begin{bmatrix} .5 & 0 \\ .5 & 1 \end{bmatrix}$	$\begin{bmatrix} .9 & 0 \\ .5 & .9 \end{bmatrix}$
$M_1 =$	$\begin{bmatrix} .3 & 0 \\ .1 & .3 \end{bmatrix}$	$\begin{bmatrix} .7 & 0 \\ .1 & .7 \end{bmatrix}$	$\begin{bmatrix} .3 & 0 \\ .1 & .3 \end{bmatrix}$	$\begin{bmatrix} .3 & 0 \\ .1 & .3 \end{bmatrix}$	$\begin{bmatrix} .1 & 0 \\ .4 & .5 \end{bmatrix}$

done for this dissertation utilize these three lengths. The number of bootstrap replicates for the bias correction suggested by Kilian (1998) was equal to $1,000$. The number of bootstrap replications for the bootstrapped percentile points is set to be 999, which enables the direct calculation of the percentile points. The desired coverage probability is chosen as $\alpha = .9$. This allows the use of the 25th and 975th ordered value of the bootstrapped distribution as the .025 and .975 percentile for the prediction intervals, given a coverage of .9 for this interval. Since the Bonferroni technique is used to generate the bootstrap prediction regions, and $K = 2$, the desired coverage of .9 for the prediction region is obtained. Kim (2001) and (2004) uses $\alpha = .95$ and $B = 999$, causing problems with the choice of the percentile points. His code is written in the language GAUSS, which in case of a non-integer value of an array access, rounds this value to the next largest integer, and hence results in coverage, which is too conservative for the upper bound and too liberal for the lower bound. This problem can be easily avoided by choosing $B = 1999$. The effects on the results given by Kim (2001) and (2004) are negligible. For each parameter combination, the number of iterations was chosen to be 500, except for a few cases.

In this case, due to computing limitations, the number of iterations was chosen to be 100. For some parameter combinations, this number was increased to 5,000, however, no difference in the simulation results was seen.

To examine the performance of the different methods, three different measures were considered. The first, the average empirical coverage is defined as

$$EC(PR) = \text{card}\{Y_{n+h}(i) : Y_{n+h}(i) \in PR\}/N,$$

where PR denotes the prediction region, card indicates "the number of" and $Y_{n+h}(i)$ is the i-th true value, generated using the true parameters and innovations generated from the true distribution. The number of generated true values, N, was chosen as 1,000, compared to 100 used by Kim (2001) and (2004). The second is the variation of average coverage of the empirical coverage of the prediction regions. The third measure used is the area of the prediction region, which is calculated by multiplying the lengths of the corresponding prediction intervals.

7. RESULTS OF THE SIMULATION STUDY

7.1. GENERAL OBSERVATIONS

The observations of the simulations vary for different methods and different parameter combinations. One general observation is that, as the length of the observed time series increases the coverage probabilities seem to get closer to the nominal coverage. In addition, the variation within the observed coverage probabilities decreases. This behavior seems natural, since it is possible to estimate more innovations and thus resample from a population, which is closer to the true population distribution of the innovations. As the prediction horizon increases, thus for larger h, the average area of the simultaneous prediction intervals increases. This increase, however, is not the same for all parameter combinations. For parameter combinations with either unit roots or roots close to unity, the increase in the area seems to be increasing at a steady rate. For parameter combinations of the stationary time series, it almost seems that the average area levels off. Both the backward and the forward representation have parameter combinations for which the coverage is not acceptable, i. e. either too liberal or too conservative. In addition, it is not possible to say that one method gives coverage that is so close to nominal such that this one method should be used. A possible distinction between the two methods could have been the average area of the simultaneous prediction intervals. The area, however, seems to correspond with the coverage probabilities, i.e., the larger the coverage is the larger is the area. So it is not possible to find a difference between the two methods when considering the average area.

Comparing the results of the simulation studies done in this dissertation with the results provided by Kim (2001) and (2004) it is seen that the results are similar. However, coverage for the percentile-t intervals is slightly more liberal than the average coverage provided by Kim. All functions in FORTRAN necessary for this method were checked against the appropriate functions provided by the GAUSS program and they matched up to the 4-th decimal. One possibility for the difference is rounding error, which could occur in either program differently. It was not possible to compare all the results considered in the simulation study for this dissertation with results found by Kim since he only published a subset of the results he found. In addition, Kim (2001) did not follow the steps suggested by Killian (1998) for the bias correction. His simultaneous intervals were more conservative than the results shown here, however, using the method used by Kim (2001) the FORTRAN code gives similar average coverage probabilities. In the following sections several graphs of coverage probabilities will be shown. Tables with all the results are provided in Appendix B.

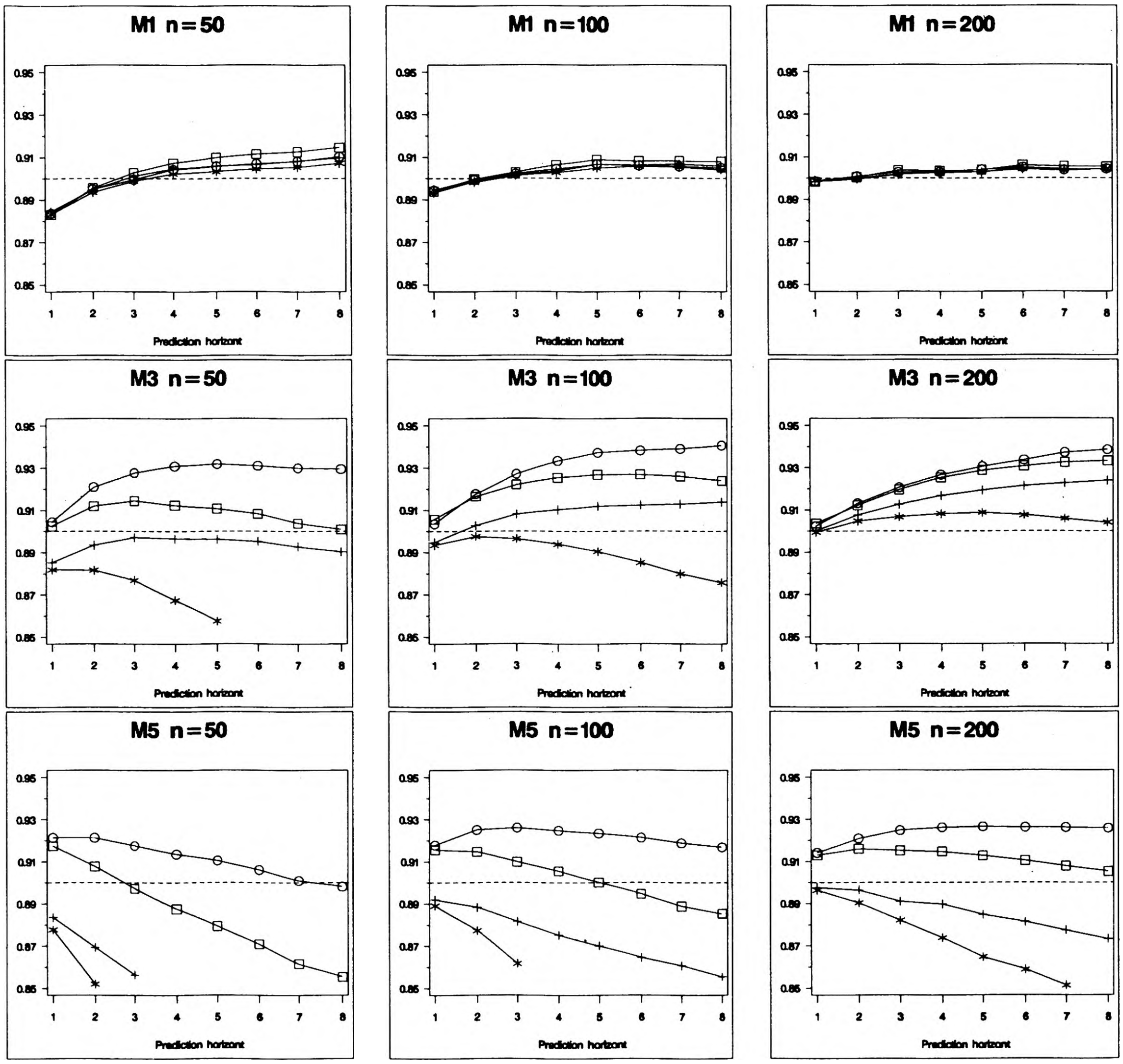
7.2. A CLOSER LOOK AT THE RESULTS

The results will be presented in subsections ordered by the order of the time series model. In some cases it may be possible to specify the order of the model. Therefore the VAR(1) and VAR(2) models will be considered first. However, in most cases it is not known what the exact order of the model is. Therefore a simulation was run where a VAR(2) was generated and AIC was used to determine the order of the time series. In addition a VARMA(1, 1) model was simulated and a VAR(p) time series was fitted where p was determined by AIC. Models with non-zero mean and inflated variance where also considered. These results complete the section on simulation results. In the following, the model used for simultaneous bootstrap prediction intervals based on the forward or backward representation of the time series will be denoted by the forward or backward model, respectively.

In general, the prediction interval for all four distribution functions of the innovations the method based on the method suggested by Pascual, Romo, and Ruiz (2004) are more conservative than the simultaneous prediction intervals based on the backward representation for stationary models. When the model is close to a unit root or has a unit root the average coverage probabilities of the backward model are too liberal whereas the coverage probabilities of the forward model are close to the nominal value. The forward representation with the bias correction of Kilian is the only model, which gives appropriate coverage when a random walk is considered. The percentile-t intervals give average coverage probabilities which are below but close to nominal. As mentioned in the introduction, the area for stationary models levels off as the prediction horizon, h, increases, which can be seen in Figure 7.5. An explanation is that, since the model is stationary, each value of the time series can be represented as a random variable with mean zero and a certain variance. However, since the prediction intervals are based on the last p observations the intervals are not centered around zero. However, as the prediction horizon gets larger, the intervals get as large due to the need to cover $(1-\alpha)100$

percent of the future values under increasing uncertainty. In case of the models close to unit root and models with unit root, as t increases the variance increases as well. Hence, the simultaneous prediction intervals must increase in area to capture $(1-\alpha)100\%$ of the values. This explains the difference in the behavior of the area. One note about the variation of the coverage probabilities, the simultaneous prediction intervals based on the bias correction proposed by Nicholls and Pope (1988) seem to have more variation in observed coverage probabilities than the intervals base on the bootstrap-after-bootstrap method. One possible explanation is that the bias correction of Nicholls and Pope is calculated for each bootstrapped time series, whereas the bootstrap-after-bootstrap bias correction is only calculated once for each generated time series.

7.2.1. Results for VAR(1). For model M1 the percentile t intervals are slightly liberal, with the percentile intervals are below nominal coverage for $h = 1$, increasing above nominal for $h \geq 2$, but staying below .916. For the model M2 the intervals are more conservative, making the coverage of the percentile t-intervals above nominal for $T = 100$ and $T = 200$. For the model with a root close to unity and with one unit root, the percentile t method gives coverage which is too liberal for small T whereas there is good coverage for $T = 200$. For the percentile method the prediction interval based on the backward model and Nicholls and Pope's bias correction gets too liberal for large h. The other three models remain close to nominal coverage, with the bootstrap-after-bootstrap forward model being too conservative with coverage close to .94. For the random walk the percentile-t method

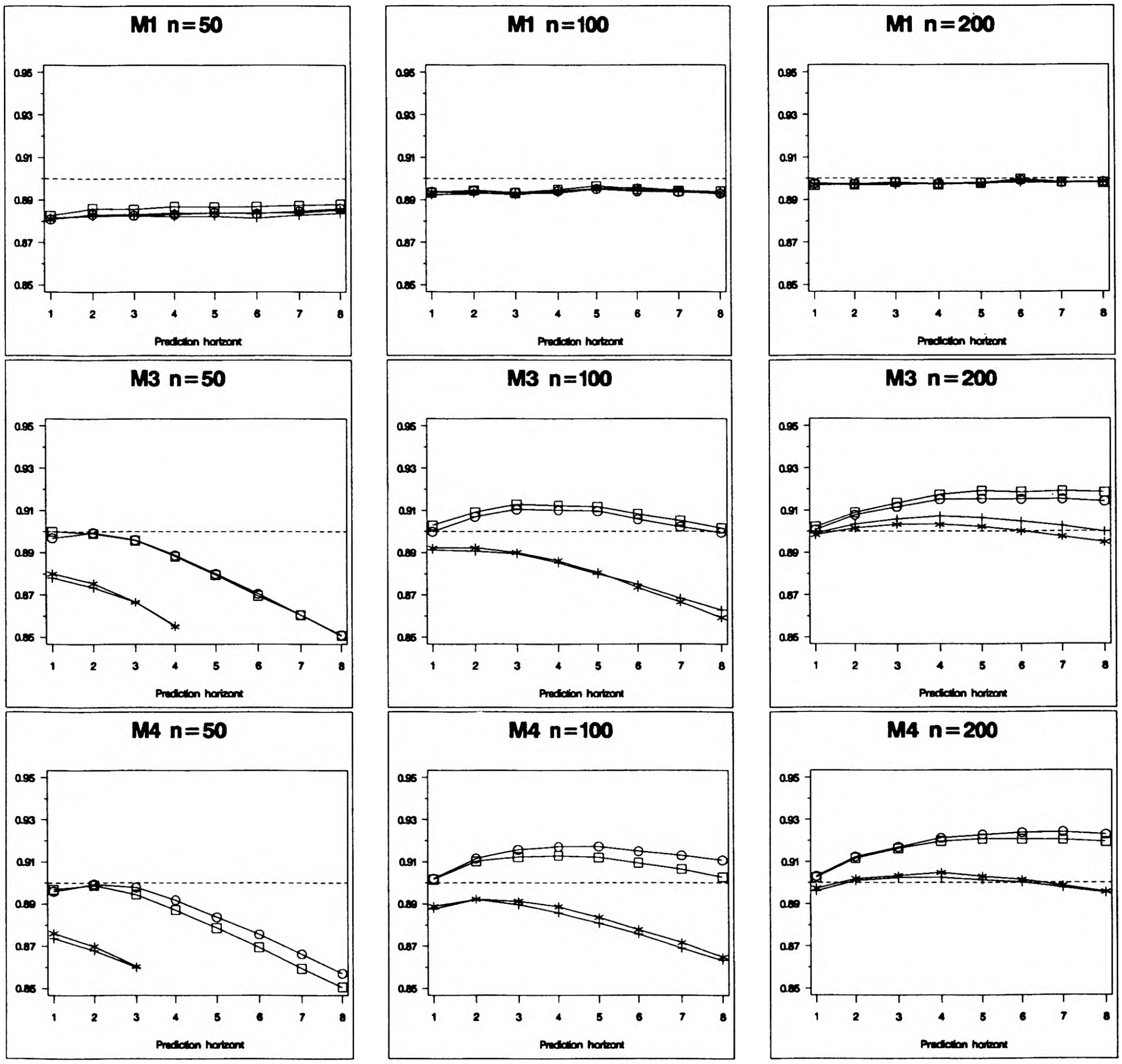


○ Forward Method Kilian □ Forward Method Nicholls and Pope
+ Backward Method Kilian $*$ Backward Method Nicholls and Pope
Note: values not on the graph are below .85.

Figure 7.1. Average Coverage Probabilities for percentile prediction intervals with normal innovations

is too liberal for large h as well as the prediction intervals based on the backward representation.

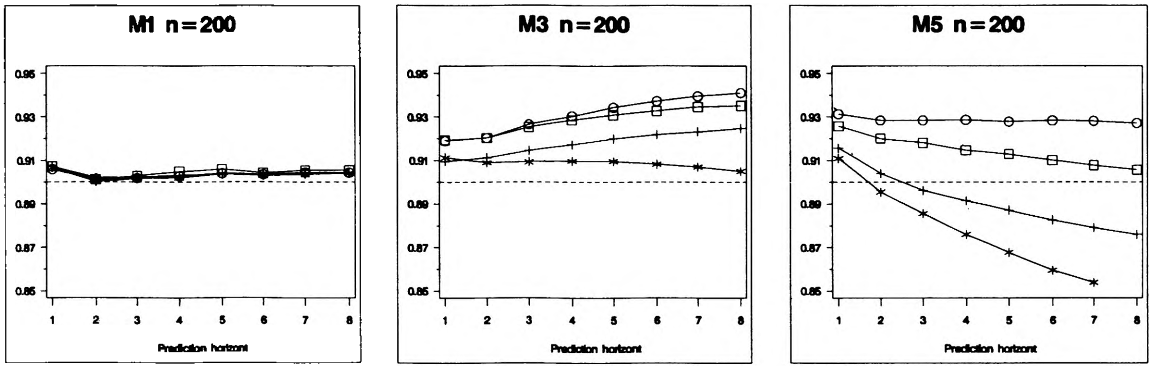
For innovations from the χ^2-distribution, for $h = 1$ the percentile-t method gives liberal coverage that increases significantly closer to nominal when $h = 2$ for



○ Forward Method Kilian □ Forward Method Nicholls and Pope
+ Backward Method Kilian * Backward Method Nicholls and Pope
Note: values not on the graph are below .85.

Figure 7.2. Average Coverage Probabilities for percentile-t prediction intervals with normal innovations

all models as can be seen in Figure 7.3. For the model M1, the percentile method has a dip, still above nominal coverage, for $h = 2$. For model M2 the percentile prediction intervals for the backward model using Nicholls and Pope's bias correction has coverage closest to nominal, whereas the other percentile prediction intervals are

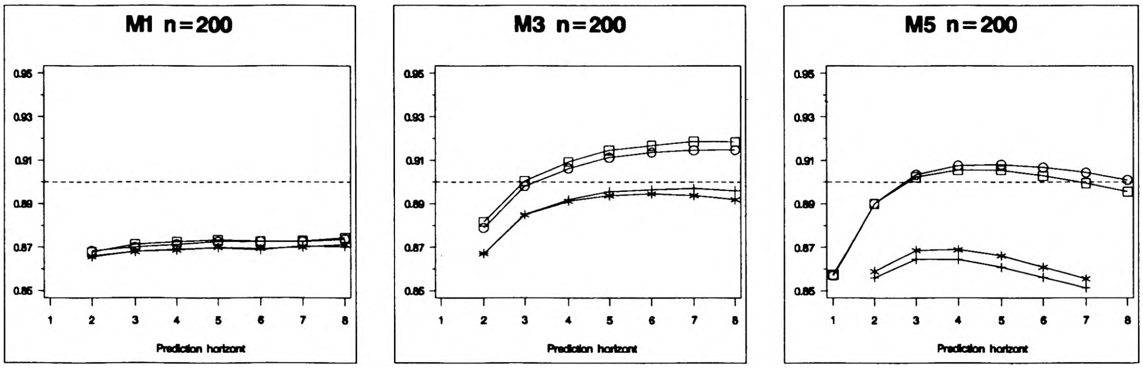


○ Forward Method Kilian □ Forward Method Nicholls and Pope
+ Backward Method Kilian $*$ Backward Method Nicholls and Pope
Note: values not on the graph are below .85.

Figure 7.3. Average Coverage Probabilities for percentile prediction intervals with χ^2 innovations

too conservative. This is similar for the models with unit roots and roots close to unity, but the backward models are too liberal for the random walk model. The results for the exponential innovations are similar to the ones of the χ^2-distribution. Therefore details are omitted. The results for the t-distribution are similar to the results for the normal innovations so details are omitted.

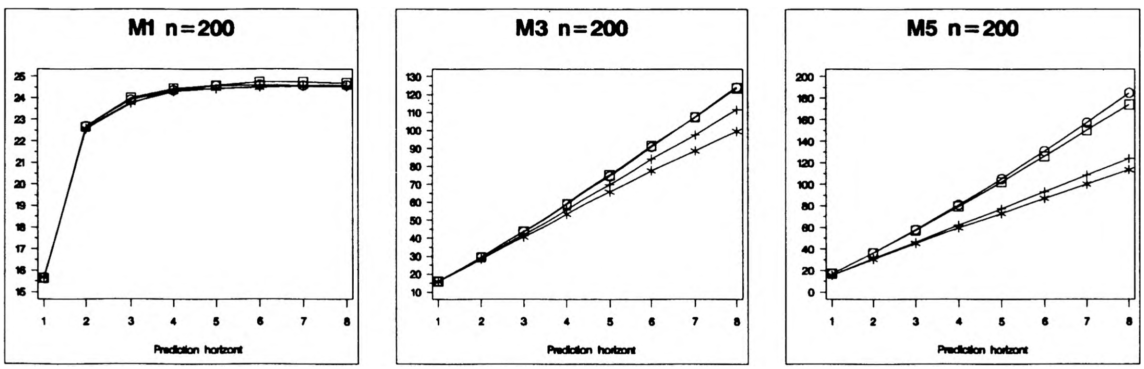
7.2.2. Results for VAR(2). For the VAR(2) models it should be noted that even though all models are stationary, model M9 has a root close to unity, which influences the coverage probabilities. First considering the models with normal innovations, for models M6 through M8 the average coverage probabilities for the percentile method are below nominal coverage for small h and then increase to about .92. The percentile t intervals are below nominal coverage for models M6 and M7, but for large T they provide slightly above nominal coverage for large h. Similar to the results for the random walk, the only method providing average



o	Forward Method Kilian	□	Forward Method Nicholls and Pope
+	Backward Method Kilian	*	Backward Method Nicholls and Pope

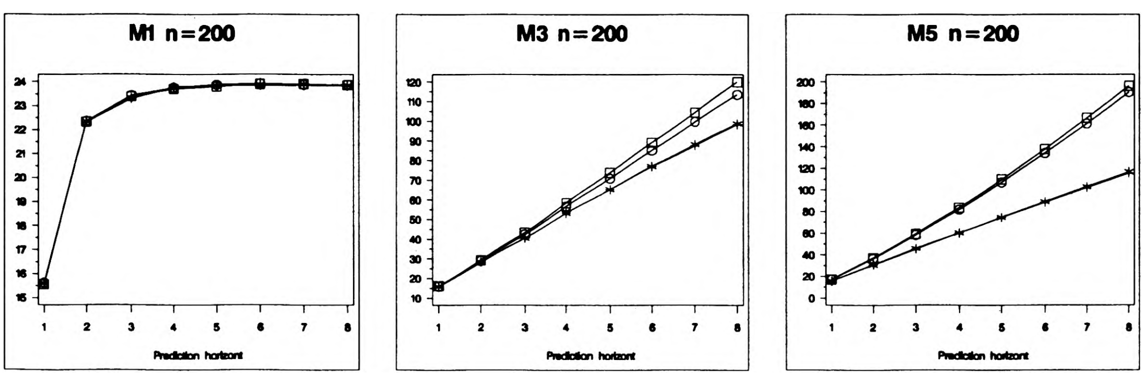
Note: values not on the graph are below .85.

Figure 7.4. Average Coverage Probabilities for percentile-t prediction intervals with χ^2 innovations



o	Forward Method Kilian	□	Forward Method Nicholls and Pope
+	Backward Method Kilian	*	Backward Method Nicholls and Pope

Figure 7.5. Average Area for percentile prediction intervals with normal innovations



o	Forward Method Kilian	□	Forward Method Nicholls and Pope
+	Backward Method Kilian	*	Backward Method Nicholls and Pope

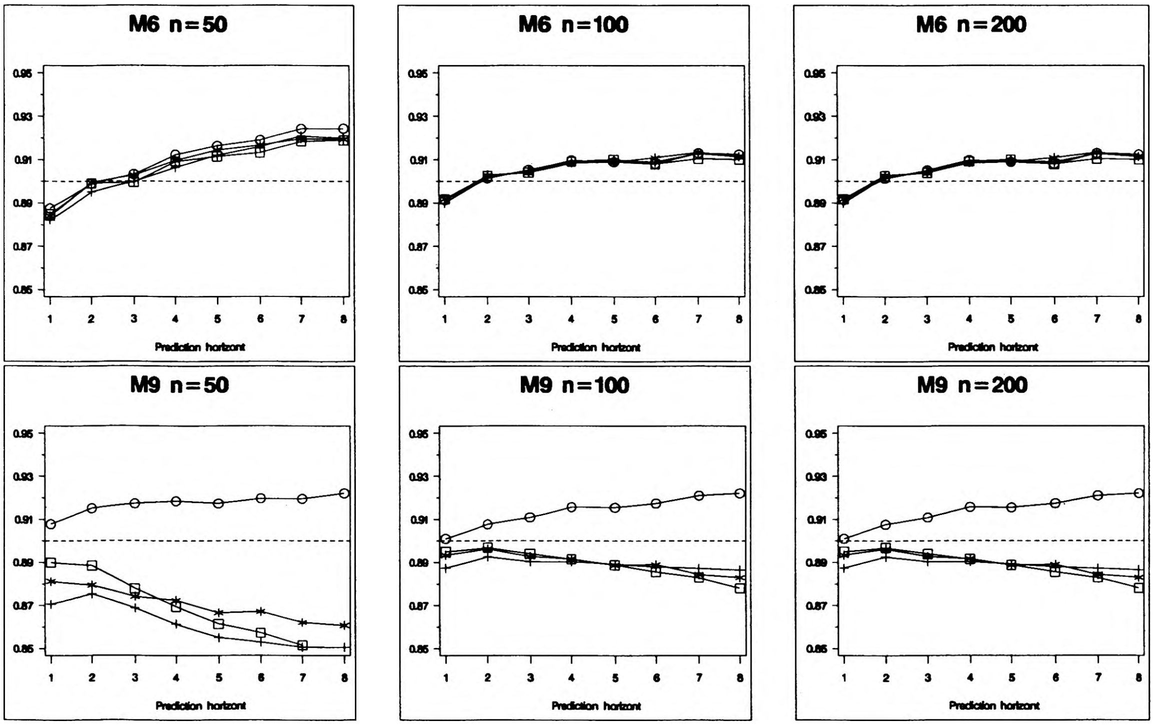
Figure 7.6. Average Area for percentile-t prediction intervals with normal innovations

coverage probabilities above nominal is the forward representation using Killian's bias correction. Unlike for the VAR(1) the coverage for the other three percentile intervals is still close to nominal coverage, as seen in Figure 7.7. For the percentile t method the average coverage probabilities are too liberal for model M9 when h is large. As seen in the VAR(1) case for stationary models the average area of the prediction intervals levels off, however for models with roots close to unity the areas continue to increase as h increases. The variance of the coverage probabilities seem to decrease as h decrease for small T, however it is almost level for large T.

For the non-symmetrical distributions the results are similar to the VAR(1) models. Again the percentile t intervals do not give coverage close to nominal. The average coverage probabilities for models with t-distributed innovations are similar to the average coverage probabilities of models with normal innovations.

7.2.3. Results for VAR(2) and VARMA(1, 1) using AIC to determine the order of the model. The four models for the VAR(2) with normal innovations were simulated and instead of assuming the order of the model equaling two, AIC was used to determine the order. Overall there is not much difference to the case where the order of the model is assumed to be fixed at two. Again for M6-M8 the coverage for the percentile method is above nominal coverage whereas for the percentile t method the coverage is below. As h increases the variance decreases. For model M9 the only method above nominal coverage is the percentile intervals using the forward model, whereas the other models are below nominal coverage.

The method proposed here is not suitable for finding prediction intervals for vector autoregressive moving average process. The method underestimates the

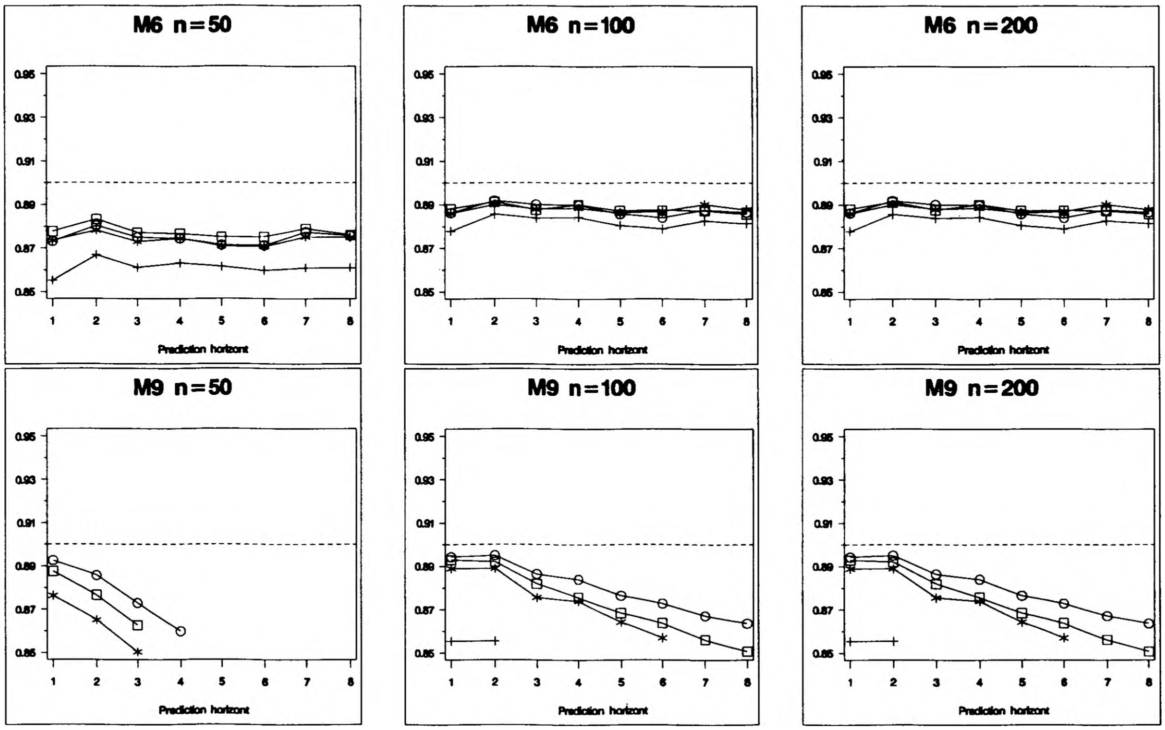


o	Forward Method Kilian	□	Forward Method Nicholls and Pope
+	Backward Method Kilian	*	Backward Method Nicholls and Pope

Note: values not on the graph are below .85.

Figure 7.7. Average Coverage Probabilities for percentile prediction intervals with normal innovations

width of the prediction intervals leading to simultaneous prediction intervals that are too liberal. Since the maximal order of the vector autoregressive time series was set to equal six, it may be possible to increase the accuracy of those prediction intervals by enabling higher order models. Since the bootstrap prediction intervals utilizing only the forward representation of time series can be applied to VARMA models, it may be possible to find prediction intervals that give close to nominal coverage fitting the correct model. This can be done by estimating all the parameters of the VARMA(p, q) process and using all estimates to find the simultaneous



○ Forward Method Kilian □ Forward Method Nicholls and Pope
+ Backward Method Kilian ∗ Backward Method Nicholls and Pope
Note: values not on the graph are below .85.

Figure 7.8. Average Coverage Probabilities for percentile-t prediction intervals with normal innovations

bootstrap prediction intervals instead of approximating the VARMA(p, q) process by a VAR(p) process.

7.2.4. Results for VAR(1) and VAR(2) with non-zero mean and inflated variance. For Gaussian innovations the simulation was run with an inflated variance and with mean equal to a vector of ones. Kim (2001) and (2004) ran simulations with non-zero mean, however he did not run a simulation with inflated variance. He states that the results for the non-zero mean were similar to the case of zero mean.

For VAR(1) with $T = 50$ it is observable that the backward model is not as liberal as for the case with a zero mean vector. For model M3 the backward method with Nicholls and Pope's bias correction becomes too liberal as h increases. The forward method is too conservative for the model M2. Again the interesting fact that for stationary models the coverage is below nominal for small h and increases above nominal as h increases. The percentile t method gives too liberal coverage for non-stationary models and models close to non-stationarity for the backward model. For the forward model the coverage is good, however slightly below nominal coverage. Again for M3 the backward method with Nicholls and Pope's bias correction becomes too liberal as h increases. Similar results as for $T = 50$ are seen for the percentile-t-method. The percentile t method gives coverage closest to nominal for $T = 200$, when the forward method is used.

For the VAR(2) for the models M6-M8 the coverage for the percentile method is too conservative and the coverage for the percentile-t-method is too liberal. Again as h increases the average coverage increases as well. For the percentile method there seems to be no difference in the average coverage probabilities between the forward and backward model. For model M9, except for the forward method using Killian's bias correction, the simultaneous prediction intervals are far too liberal for $h > 3$. This model has a root close to unity. Similar observations can be made for $T = 100$ and $T = 200$, but again the coverage probabilities are closer to nominal coverage. For $T = 200$, the backward model for percentile t intervals with Killian's bias correction is too liberal all the models.

For stationary VAR(1) models with inflated variance, the coverage for the percentile method is slightly above nominal coverage. For the percentile t method it is slightly below. For a model with roots close to unity the average coverage probabilities for the percentile t method are too liberal for small T but is closer to nominal coverage for $T = 200$ than the percentile method. For the percentile method the coverage is too conservative for large h with coverage probabilities close to .93 for $T = 200$. For models with one unit root the percentile t method is too liberal for $T = 50$, however for $T = 200$ the backward model gives the best coverage. The percentile method, giving almost nominal coverage for $T = 50$, becomes conservative for $T = 200$. However, for the random walk the percentile intervals using forward model give the coverage that is closest to nominal, whereas the other models are too liberal.

For the VAR(2) model, inflating the variance does not change the average coverage probabilities compared to the model without inflated variance. For models M6-M8, the coverage probabilities increase as h increases, and are right around nominal coverage. As T increases the coverage is closer to nominal coverage. Also for model M9, it is similar with only the forward representation using Killian's bias correction being above nominal coverage, whereas the other models as h increases are too liberal with coverage below .87.

There is not much difference when inflating the variance and changing the mean for the innovations compared to the model described in Sections 7.2.1. and 7.2.2.. This makes the method applicable even if the mean is not zero or has inflated variance without correcting for either.

8. CONCLUSION

In this dissertation, a method for finding simultaneous prediction intervals for vector autoregressive time series has been proposed. This was done be applying a method proposed by Pascual, Romo, and Ruiz (2004) for the univariate time series to the case of multivariate time series. Unlike the previously proposed method utilizing the backward representation of the VAR(p) time series by Kim (1999), (2001) and (2004), the method discussed in this dissertation does not require the use of the backward representation of the time series. Therefore it is applicable to models that do not have a backward representation. The asymptotic results established in papers dealing with bootstrap prediction intervals only show weak convergence of the bootstrapped random variable to the true random variable. In this dissertation, a proof is provided that weak convergence of the bootstrapped random variable implies that the bootstrapped prediction bounds converge to the true prediction bounds. The asymptotic validity of the method is established, i.e., the bootstrap random variable using the forward representation of the time series converges in distribution to the true random variable.

The asymptotic result does not provide an idea of how well the method works in small samples. A simulation study is conducted, showing that the proposed method gives almost the same results as the method using the backward representation. For models with unit roots however, the proposed method gives better results than the method proposed by Kim (2001) and (2004). The simulation study also shows that percentile t intervals do not give the required coverage if the innovations do not have a symmetrical distribution.

The proposed method has several applications such as simultaneously predicting dependent financial data. Especially, since the proposed method works for the random walk, it can be applied to modeling financial derivates. The implementation is easier than the implementation of the method requiring the backward representation, since only the forward representation of the time series is needed. In addition this method can be used for time series that do not have a backward representation, or the representation is difficult to find.

9. FUTURE RESEARCH

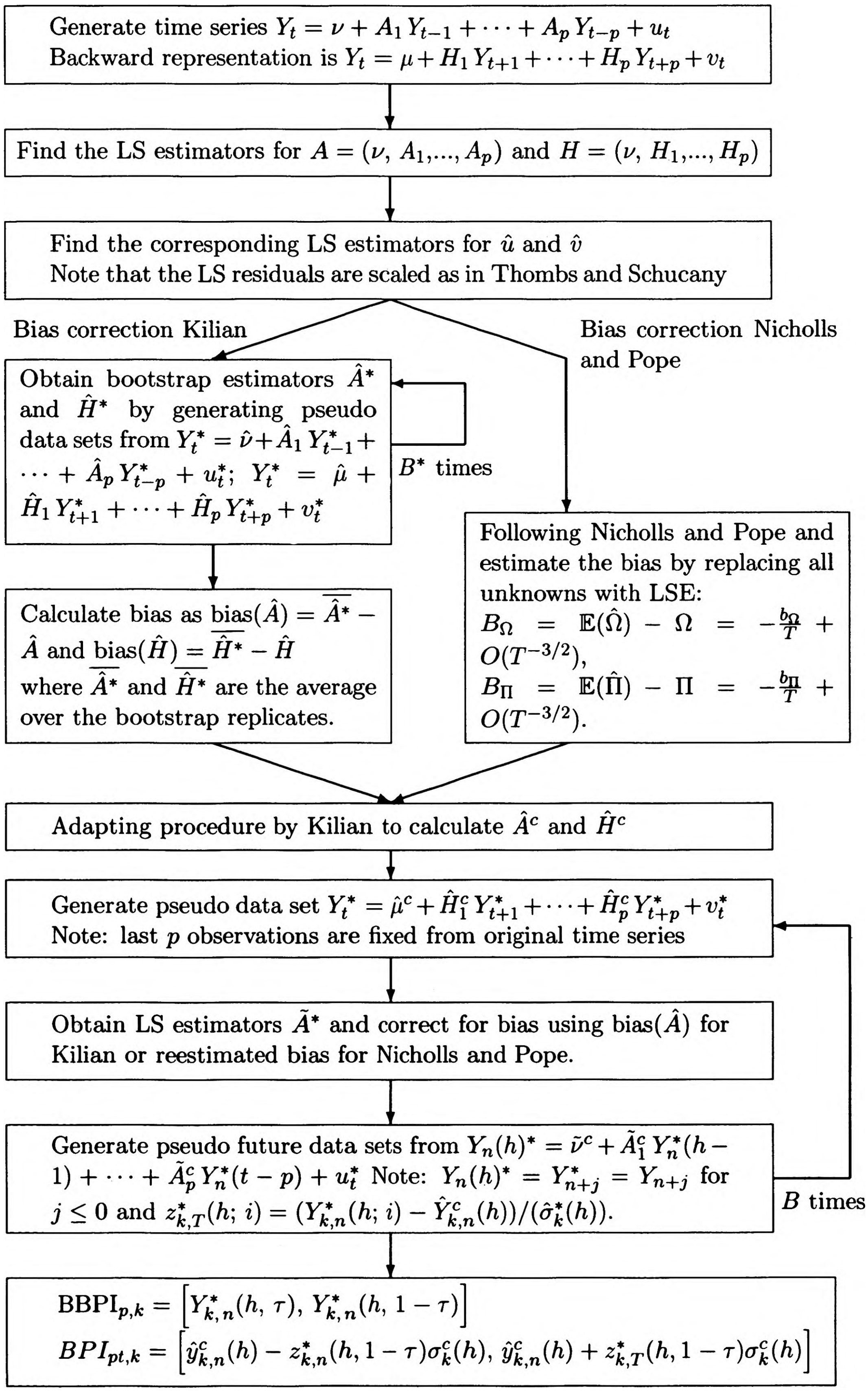
In this section future research will be described. Since it was not possible to show a difference between the backward and forward prediction intervals it is of interest to look at the speed of convergence of the two methods. It is also interesting to be able to compare the bootstrap prediction intervals to the normal approximation intervals. Due to limitation of the IMSL libraries for estimators of the parameters in FORTRAN it was not possible to write the code for VARMA time series. A simulation study can show whether bootstrap prediction interval for VARMA time series give good sample coverage. In addition it will be necessary to show that the bootstrapped estimators of VARMA time series converge to the true parameters. Another major area of interest is to apply the bootstrap method for finding prediction intervals to the case of models with exogenous (uncorrelated with the disturbances) variables. This may help in finding better prediction intervals for applications such as predicting simultaneous stocks prices, since variables other than the past stock price influence the price of the stock in the future.

To find simultaneous prediction intervals, the method suggested by Bonferroni is used. However in many applications these intervals are too conservative. It is necessary to examine the behavior if more than two simultaneous intervals are calculated. If in this case the Bonferroni method gives too conservative results it may be possible to improve the coverage probabilities by using different methods for finding prediction intervals.

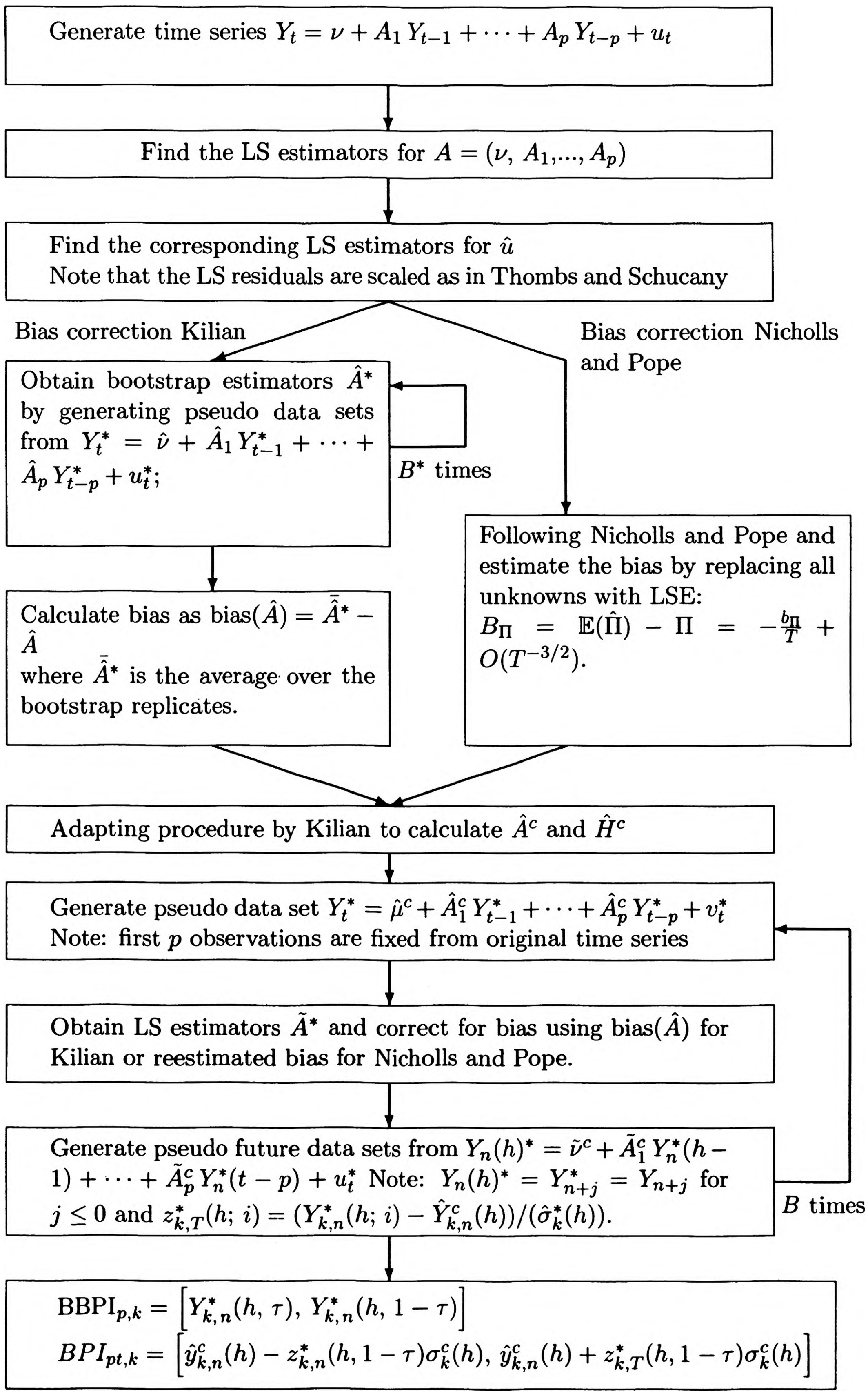
APPENDIX A.

FLOW CHARTS OF FORTRAN CODE

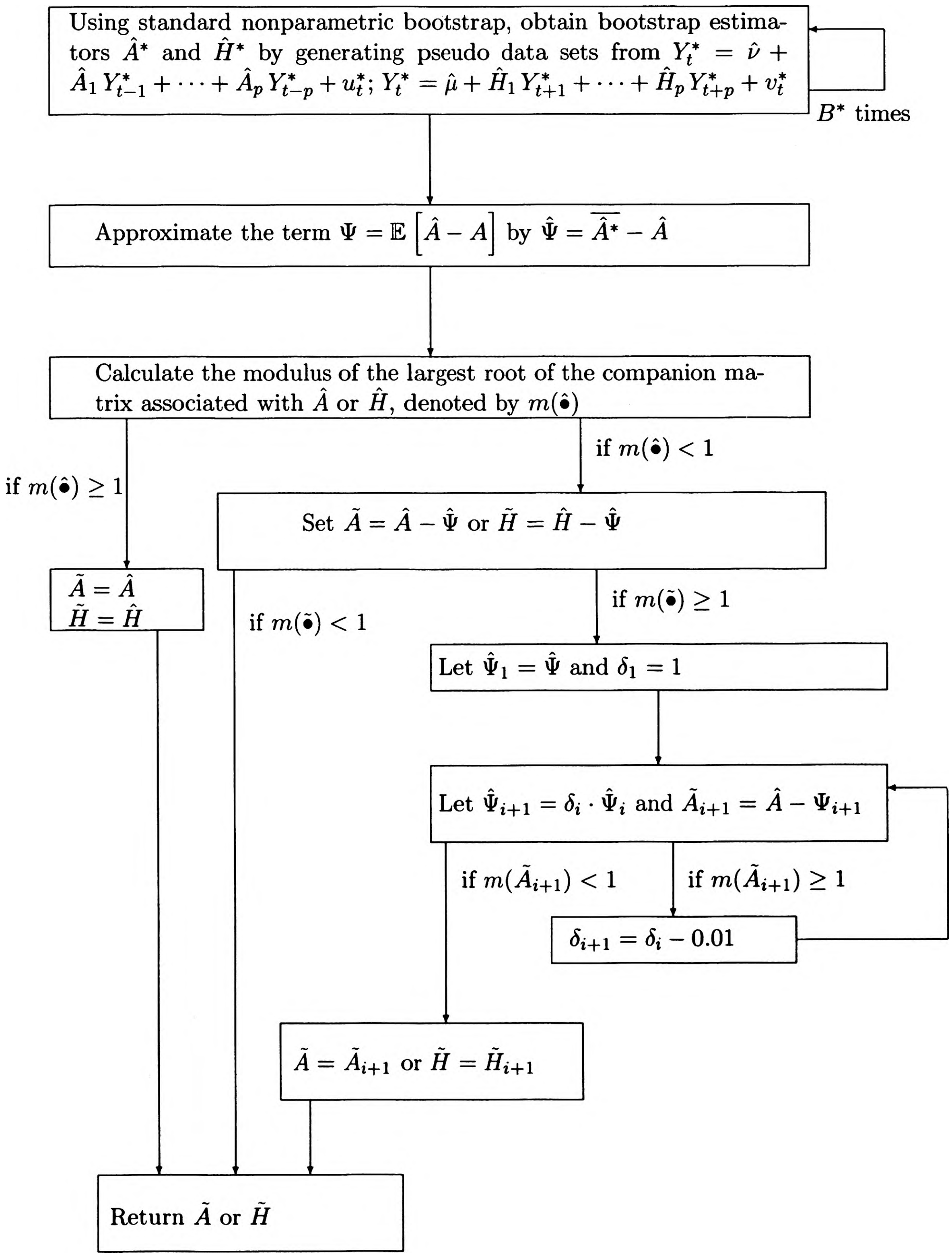
Flow Chart of Program using backward representation and Kilian



Flow Chart of Program using Forward Representation and Kilian



Flow Chart for ensuring Stationarity



APPENDIX B.

TABLES OF RESULTS

Legend for Tables

PRR	Percentile intervals using forward method using Kilian's bias correction
PRRP	Percentile intervals using forward method using Nicholls and Pope's bias correction
KIM	Percentile intervals using backward method using Kilian's bias correction
KIMP	Percentile intervals using backward method using Nicholls and Pope's bias correction
PRR%	Percentile-t intervals using forward method using Kilian's bias correction
PRRP%	Percentile-t intervals using forward method using Nicholls and Pope's bias correction
KIM%	Percentile-t intervals using backward method using Kilian's bias correction
KIMP%	Percentile-t intervals using backward method using Nicholls and Pope's bias correction
Nor	Innovations: Normal distribution
χ^2	Innovations: χ^2- distribution
Exp	Innovations: Exponential distribution
t-dist	Innovations: Student-t distribution

Table B.1. Average Coverage Probability for Model M1 T=50

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.8837	0.8950	0.8995	0.9042	0.9058	0.9069	0.9082	0.9104
PRRP:	0.8829	0.8956	0.9028	0.9074	0.9099	0.9115	0.9127	0.9148
KIM:	0.8845	0.8948	0.9012	0.9047	0.9060	0.9065	0.9083	0.9098
KIMP:	0.8832	0.8935	0.8988	0.9024	0.9033	0.9044	0.9053	0.9072
PRR%:	0.8810	0.8833	0.8827	0.8832	0.8840	0.8836	0.8847	0.8861
PRRP%:	0.8827	0.8861	0.8856	0.8869	0.8866	0.8869	0.8874	0.8881
KIM%:	0.8816	0.8825	0.8824	0.8822	0.8824	0.8815	0.8828	0.8838
KIMP%:	0.8809	0.8833	0.8831	0.8840	0.8839	0.8840	0.8840	0.8854

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9011	0.8946	0.9006	0.9013	0.9038	0.9052	0.9054	0.9060
PRRP:	0.9006	0.8962	0.9019	0.9041	0.9071	0.9083	0.9088	0.9099
KIM:	0.9028	0.8951	0.9004	0.9021	0.9039	0.9048	0.9063	0.9064
KIMP:	0.9041	0.8944	0.8983	0.9002	0.9027	0.9036	0.9043	0.9041
PRR%:	0.8491	0.8685	0.8707	0.8703	0.8719	0.8719	0.8734	0.8729
PRRP%:	0.8498	0.8699	0.8731	0.8735	0.8743	0.8748	0.8758	0.8755
KIM%:	0.8361	0.8581	0.8600	0.8605	0.8613	0.8604	0.8628	0.8617
KIMP%:	0.8365	0.8593	0.8618	0.8612	0.8622	0.8622	0.8640	0.8631

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9170	0.8977	0.9025	0.9034	0.9052	0.9078	0.9082	0.9083
PRRP:	0.9171	0.8980	0.9040	0.9056	0.9101	0.9103	0.9112	0.9127
KIM:	0.9194	0.8972	0.9020	0.9030	0.9066	0.9070	0.9078	0.9082
KIMP:	0.9215	0.8979	0.9024	0.9021	0.9045	0.9061	0.9060	0.9063
PRR%:	0.8364	0.8626	0.8659	0.8665	0.8686	0.8697	0.8698	0.8701
PRRP%:	0.8366	0.8651	0.8679	0.8688	0.8721	0.8719	0.8722	0.8733
KIM%:	0.8170	0.8489	0.8510	0.8504	0.8536	0.8542	0.8535	0.8545
KIMP%:	0.8169	0.8498	0.8523	0.8522	0.8542	0.8548	0.8550	0.8555

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8812	0.8924	0.8955	0.8993	0.9001	0.9019	0.9023	0.9033
PRRP:	0.8815	0.8916	0.8968	0.9003	0.9033	0.9044	0.9043	0.9054
KIM:	0.8820	0.8920	0.8962	0.8999	0.9017	0.9031	0.9028	0.9035
KIMP:	0.8839	0.8911	0.8941	0.8965	0.8987	0.8993	0.8998	0.9011
PRR%:	0.8915	0.8925	0.8917	0.8923	0.8926	0.8915	0.8919	0.8928
PRRP%:	0.8926	0.8927	0.8927	0.8933	0.8940	0.8928	0.8926	0.8942
KIM%:	0.8861	0.8858	0.8853	0.8869	0.8866	0.8861	0.8853	0.8858
KIMP%:	0.8881	0.8878	0.8865	0.8875	0.8876	0.8871	0.8876	0.8887

Table B.2. Average Coverage Probability for Model M1 T=100

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.8945	0.8994	0.9023	0.9045	0.9069	0.9061	0.9055	0.9051
PRRP:	0.8940	0.8998	0.9030	0.9064	0.9090	0.9082	0.9082	0.9080
KIM:	0.8932	0.8987	0.9018	0.9036	0.9068	0.9064	0.9066	0.9059
KIMP:	0.8947	0.8982	0.9013	0.9029	0.9050	0.9057	0.9053	0.9042
PRR%:	0.8939	0.8943	0.8935	0.8943	0.8952	0.8940	0.8937	0.8930
PRRP%:	0.8934	0.8946	0.8933	0.8949	0.8966	0.8952	0.8942	0.8939
KIM%:	0.8924	0.8930	0.8930	0.8934	0.8951	0.8946	0.8942	0.8927
KIMP%:	0.8936	0.8935	0.8923	0.8942	0.8953	0.8959	0.8944	0.8934

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9083	0.9033	0.9068	0.9083	0.9098	0.9099	0.9096	0.9111
PRRP:	0.9092	0.9045	0.9087	0.9098	0.9118	0.9128	0.9120	0.9132
KIM:	0.9098	0.9042	0.9073	0.9087	0.9092	0.9100	0.9096	0.9109
KIMP:	0.9102	0.9031	0.9060	0.9079	0.9090	0.9087	0.9101	0.9104
PRR%:	0.8377	0.8725	0.8757	0.8770	0.8782	0.8780	0.8784	0.8799
PRRP%:	0.8385	0.8728	0.8770	0.8788	0.8804	0.8797	0.8792	0.8814
KIM%:	0.8296	0.8673	0.8707	0.8713	0.8726	0.8728	0.8720	0.8739
KIMP%:	0.8297	0.8677	0.8708	0.8726	0.8742	0.8729	0.8731	0.8745

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9245	0.9052	0.9071	0.9102	0.9097	0.9103	0.9108	0.9105
PRRP:	0.9243	0.9055	0.9093	0.9117	0.9113	0.9120	0.9131	0.9122
KIM:	0.9265	0.9044	0.9080	0.9093	0.9088	0.9096	0.9105	0.9103
KIMP:	0.9294	0.9057	0.9083	0.9081	0.9081	0.9087	0.9087	0.9086
PRR%:	0.8157	0.8624	0.8649	0.8684	0.8683	0.8679	0.8688	0.8689
PRRP%:	0.8159	0.8626	0.8666	0.8693	0.8695	0.8693	0.8694	0.8697
KIM%:	0.8029	0.8544	0.8568	0.8601	0.8597	0.8595	0.8594	0.8601
KIMP%:	0.8033	0.8553	0.8580	0.8602	0.8602	0.8605	0.8603	0.8601

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8912	0.8986	0.9000	0.9017	0.9017	0.9046	0.9039	0.9069
PRRP:	0.8929	0.8989	0.9006	0.9017	0.9028	0.9064	0.9082	0.9065
KIM:	0.8930	0.8972	0.9020	0.9045	0.9019	0.9042	0.9059	0.9057
KIMP:	0.8921	0.8970	0.9021	0.9036	0.9032	0.9016	0.9063	0.9057
PRR%:	0.8996	0.8986	0.8996	0.8984	0.8995	0.9013	0.9026	0.9024
PRRP%:	0.8991	0.8992	0.8993	0.8999	0.8986	0.9022	0.9030	0.9012
KIM%:	0.8962	0.8945	0.8964	0.8974	0.8965	0.8974	0.8980	0.8978
KIMP%:	0.8958	0.8951	0.8985	0.8979	0.8976	0.8967	0.8997	0.9000

Table B.3. Average Coverage Probability for Model M1 T=200

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.8986	0.9010	0.9029	0.9030	0.9042	0.9052	0.9039	0.9045
PRRP:	0.8985	0.9005	0.9038	0.9036	0.9040	0.9062	0.9056	0.9055
KIM:	0.8985	0.8997	0.9017	0.9037	0.9028	0.9050	0.9042	0.9042
KIMP:	0.8990	0.9000	0.9019	0.9025	0.9029	0.9043	0.9036	0.9045
PRR%:	0.8976	0.8974	0.8982	0.8976	0.8981	0.8992	0.8980	0.8984
PRRP%:	0.8970	0.8972	0.8983	0.8971	0.8978	0.8998	0.8983	0.8980
KIM%:	0.8975	0.8969	0.8971	0.8976	0.8978	0.8983	0.8979	0.8981
KIMP%:	0.8972	0.8975	0.8976	0.8977	0.8976	0.8991	0.8983	0.8984

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9059	0.9011	0.9019	0.9028	0.9040	0.9035	0.9045	0.9044
PRRP:	0.9073	0.9012	0.9030	0.9049	0.9059	0.9043	0.9056	0.9056
KIM:	0.9072	0.9021	0.9023	0.9032	0.9035	0.9036	0.9041	0.9041
KIMP:	0.9067	0.9002	0.9018	0.9022	0.9037	0.9030	0.9035	0.9042
PRR%:	0.8249	0.8684	0.8700	0.8714	0.8727	0.8726	0.8726	0.8737
PRRP%:	0.8254	0.8679	0.8713	0.8725	0.8735	0.8727	0.8729	0.8744
KIM%:	0.8206	0.8661	0.8681	0.8691	0.8698	0.8689	0.8706	0.8703
KIMP%:	0.8200	0.8656	0.8680	0.8688	0.8701	0.8693	0.8701	0.8713

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9231	0.9035	0.9051	0.9049	0.9062	0.9060	0.9063	0.9066
PRRP:	0.9235	0.9043	0.9057	0.9057	0.9080	0.9077	0.9080	0.9083
KIM:	0.9244	0.9043	0.9051	0.9059	0.9067	0.9050	0.9053	0.9058
KIMP:	0.9247	0.9029	0.9039	0.9050	0.9054	0.9048	0.9049	0.9057
PRR%:	0.7992	0.8529	0.8579	0.8598	0.8591	0.8606	0.8608	0.8604
PRRP%:	0.7995	0.8539	0.8583	0.8603	0.8600	0.8611	0.8615	0.8616
KIM%:	0.7901	0.8489	0.8523	0.8551	0.8548	0.8548	0.8553	0.8556
KIMP%:	0.7902	0.8488	0.8526	0.8555	0.8548	0.8554	0.8552	0.8558

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8983	0.9030	0.9027	0.9032	0.9035	0.9042	0.9044	0.9048
PRRP:	0.8986	0.9025	0.9039	0.9044	0.9044	0.9054	0.9054	0.9054
KIM:	0.8982	0.9030	0.9028	0.9036	0.9055	0.9060	0.9047	0.9057
KIMP:	0.8977	0.9022	0.9029	0.9039	0.9042	0.9046	0.9045	0.9047
PRR%:	0.9013	0.9039	0.9023	0.9025	0.9021	0.9031	0.9030	0.9033
PRRP%:	0.9021	0.9030	0.9027	0.9026	0.9018	0.9036	0.9032	0.9034
KIM%:	0.8996	0.9026	0.9004	0.9006	0.9018	0.9029	0.9018	0.9024
KIMP%:	0.8994	0.9012	0.9010	0.9018	0.9016	0.9022	0.9021	0.9020

Table B.4. Average Coverage Probability for Model M2 T=50

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.8861	0.9002	0.9059	0.9124	0.9153	0.9170	0.9190	0.9212
PRRP:	0.8854	0.8986	0.9029	0.9082	0.9124	0.9124	0.9143	0.9161
KIM:	0.8861	0.9005	0.9064	0.9119	0.9163	0.9176	0.9197	0.9221
KIMP:	0.8848	0.8918	0.8939	0.8957	0.8982	0.8985	0.8987	0.9000
PRR%:	0.8846	0.8855	0.8816	0.8803	0.8800	0.8786	0.8790	0.8798
PRRP%:	0.8874	0.8889	0.8860	0.8846	0.8849	0.8831	0.8834	0.8841
KIM%:	0.8816	0.8816	0.8784	0.8747	0.8743	0.8738	0.8729	0.8731
KIMP%:	0.8824	0.8797	0.8766	0.8729	0.8740	0.8724	0.8723	0.8740

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9068	0.9094	0.9147	0.9195	0.9238	0.9256	0.9274	0.9296
PRRP:	0.9062	0.9084	0.9134	0.9175	0.9213	0.9234	0.9243	0.9260
KIM:	0.9055	0.9088	0.9139	0.9181	0.9228	0.9251	0.9264	0.9283
KIMP:	0.9054	0.9016	0.9026	0.9047	0.9065	0.9075	0.9086	0.9097
PRR%:	0.8517	0.8714	0.8752	0.8768	0.8786	0.8785	0.8799	0.8815
PRRP%:	0.8533	0.8731	0.8781	0.8795	0.8819	0.8824	0.8834	0.8849
KIM%:	0.8356	0.8551	0.8588	0.8595	0.8615	0.8612	0.8617	0.8620
KIMP%:	0.8357	0.8525	0.8550	0.8550	0.8573	0.8578	0.8587	0.8600

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9192	0.9179	0.9185	0.9215	0.9231	0.9252	0.9263	0.9285
PRRP:	0.9205	0.9168	0.9175	0.9194	0.9211	0.9227	0.9240	0.9245
KIM:	0.9187	0.9176	0.9175	0.9205	0.9240	0.9255	0.9271	0.9290
KIMP:	0.9185	0.9120	0.9080	0.9092	0.9086	0.9095	0.9110	0.9114
PRR%:	0.8374	0.8611	0.8679	0.8700	0.8707	0.8710	0.8720	0.8738
PRRP%:	0.8387	0.8632	0.8711	0.8731	0.8744	0.8754	0.8764	0.8771
KIM%:	0.8136	0.8367	0.8431	0.8450	0.8459	0.8465	0.8463	0.8476
KIMP%:	0.8142	0.8360	0.8412	0.8425	0.8424	0.8433	0.8444	0.8444

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8854	0.8965	0.8996	0.9030	0.9068	0.9086	0.9101	0.9124
PRRP:	0.8836	0.8937	0.8985	0.9010	0.9036	0.9047	0.9074	0.9087
KIM:	0.8850	0.8955	0.9000	0.9044	0.9069	0.9095	0.9115	0.9125
KIMP:	0.8848	0.8901	0.8903	0.8901	0.8898	0.8907	0.8916	0.8931
PRR%:	0.8944	0.8936	0.8888	0.8854	0.8850	0.8832	0.8832	0.8847
PRRP%:	0.8948	0.8942	0.8923	0.8894	0.8886	0.8879	0.8881	0.8882
KIM%:	0.8886	0.8865	0.8820	0.8788	0.8782	0.8776	0.8776	0.8774
KIMP%:	0.8899	0.8869	0.8828	0.8798	0.8783	0.8782	0.8785	0.8786

Table B.5. Average Coverage Probability for Model M2 T=100

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.8944	0.9048	0.9096	0.9126	0.9155	0.9173	0.9178	0.9187
PRRP:	0.8945	0.9036	0.9074	0.9114	0.9130	0.9153	0.9171	0.9182
KIM:	0.8949	0.9051	0.9084	0.9120	0.9143	0.9151	0.9170	0.9185
KIMP:	0.8951	0.9016	0.9032	0.9047	0.9064	0.9074	0.9068	0.9067
PRR%:	0.8927	0.8970	0.8969	0.8959	0.8958	0.8956	0.8962	0.8967
PRRP%:	0.8938	0.8972	0.8968	0.8960	0.8960	0.8963	0.8973	0.8981
KIM%:	0.8919	0.8956	0.8946	0.8935	0.8939	0.8930	0.8941	0.8948
KIMP%:	0.8918	0.8949	0.8938	0.8939	0.8947	0.8947	0.8941	0.8943

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9084	0.9116	0.9140	0.9167	0.9194	0.9200	0.9215	0.9220
PRRP:	0.9076	0.9106	0.9137	0.9156	0.9175	0.9187	0.9206	0.9197
KIM:	0.9101	0.9121	0.9153	0.9166	0.9196	0.9213	0.9225	0.9230
KIMP:	0.9085	0.9081	0.9096	0.9104	0.9111	0.9117	0.9122	0.9127
PRR%:	0.8358	0.8668	0.8743	0.8759	0.8778	0.8782	0.8792	0.8790
PRRP%:	0.8363	0.8672	0.8755	0.8768	0.8781	0.8792	0.8805	0.8797
KIM%:	0.8262	0.8575	0.8649	0.8661	0.8674	0.8688	0.8683	0.8685
KIMP%:	0.8259	0.8572	0.8640	0.8655	0.8664	0.8675	0.8673	0.8672

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9280	0.9215	0.9203	0.9226	0.9234	0.9259	0.9271	0.9276
PRRP:	0.9264	0.9200	0.9196	0.9211	0.9238	0.9245	0.9261	0.9261
KIM:	0.9283	0.9215	0.9203	0.9229	0.9242	0.9257	0.9263	0.9282
KIMP:	0.9270	0.9174	0.9155	0.9155	0.9156	0.9163	0.9171	0.9182
PRR%:	0.8163	0.8503	0.8603	0.8645	0.8658	0.8684	0.8698	0.8680
PRRP%:	0.8163	0.8510	0.8611	0.8644	0.8671	0.8689	0.8704	0.8691
KIM%:	0.8014	0.8366	0.8466	0.8502	0.8520	0.8533	0.8544	0.8533
KIMP%:	0.8012	0.8356	0.8452	0.8480	0.8492	0.8510	0.8522	0.8516

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8968	0.9041	0.9045	0.9040	0.9069	0.9078	0.9087	0.9086
PRRP:	0.8955	0.9015	0.9045	0.9041	0.9058	0.9073	0.9087	0.9070
KIM:	0.8950	0.9026	0.9044	0.9058	0.9079	0.9087	0.9093	0.9079
KIMP:	0.8963	0.8979	0.8986	0.8996	0.8979	0.8982	0.8983	0.8980
PRR%:	0.9016	0.9042	0.9021	0.8980	0.8977	0.8970	0.8990	0.8971
PRRP%:	0.9005	0.9041	0.9028	0.8990	0.8988	0.8969	0.8991	0.8965
KIM%:	0.8990	0.9018	0.8984	0.8970	0.8943	0.8949	0.8955	0.8931
KIMP%:	0.8988	0.9010	0.8976	0.8956	0.8931	0.8931	0.8933	0.8932

Table B.6. Average Coverage Probability for Model M2 T=200

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.8985	0.9054	0.9076	0.9091	0.9113	0.9111	0.9117	0.9124
PRRP:	0.8999	0.9055	0.9078	0.9092	0.9104	0.9106	0.9111	0.9122
KIM:	0.8981	0.9056	0.9088	0.9098	0.9117	0.9125	0.9122	0.9122
KIMP:	0.8992	0.9038	0.9056	0.9055	0.9060	0.9072	0.9072	0.9081
PRR%:	0.8977	0.9016	0.9011	0.9010	0.9025	0.9009	0.9014	0.9014
PRRP%:	0.8990	0.9022	0.9019	0.9013	0.9014	0.9006	0.9018	0.9016
KIM%:	0.8973	0.9021	0.9018	0.9014	0.9017	0.9015	0.9008	0.9005
KIMP%:	0.8981	0.9016	0.9012	0.9001	0.9003	0.9010	0.9017	0.9023

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9067	0.9102	0.9125	0.9129	0.9157	0.9167	0.9169	0.9177
PRRP:	0.9066	0.9108	0.9123	0.9143	0.9157	0.9165	0.9172	0.9174
KIM:	0.9070	0.9102	0.9118	0.9132	0.9151	0.9160	0.9171	0.9177
KIMP:	0.9059	0.9084	0.9082	0.9096	0.9110	0.9110	0.9120	0.9121
PRR%:	0.8253	0.8616	0.8714	0.8743	0.8745	0.8757	0.8759	0.8764
PRRP%:	0.8256	0.8620	0.8718	0.8745	0.8748	0.8759	0.8760	0.8756
KIM%:	0.8206	0.8571	0.8662	0.8694	0.8697	0.8704	0.8705	0.8709
KIMP%:	0.8202	0.8569	0.8653	0.8686	0.8692	0.8696	0.8697	0.8697

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9238	0.9168	0.9158	0.9167	0.9183	0.9196	0.9202	0.9209
PRRP:	0.9231	0.9152	0.9149	0.9163	0.9187	0.9194	0.9201	0.9211
KIM:	0.9239	0.9166	0.9166	0.9179	0.9198	0.9201	0.9212	0.9221
KIMP:	0.9242	0.9147	0.9125	0.9120	0.9138	0.9158	0.9157	0.9165
PRR%:	0.7995	0.8405	0.8526	0.8574	0.8584	0.8604	0.8607	0.8605
PRRP%:	0.8000	0.8403	0.8527	0.8572	0.8595	0.8604	0.8609	0.8610
KIM%:	0.7903	0.8319	0.8446	0.8487	0.8502	0.8514	0.8522	0.8518
KIMP%:	0.7902	0.8316	0.8435	0.8481	0.8493	0.8507	0.8512	0.8505

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8982	0.9039	0.9054	0.9075	0.9075	0.9075	0.9087	0.9086
PRRP:	0.8985	0.9034	0.9053	0.9071	0.9072	0.9076	0.9089	0.9083
KIM:	0.8990	0.9042	0.9063	0.9081	0.9073	0.9078	0.9084	0.9088
KIMP:	0.8992	0.9033	0.9041	0.9048	0.9040	0.9041	0.9059	0.9055
PRR%:	0.9029	0.9063	0.9051	0.9048	0.9044	0.9032	0.9041	0.9037
PRRP%:	0.9037	0.9054	0.9060	0.9054	0.9043	0.9044	0.9047	0.9038
KIM%:	0.9020	0.9044	0.9044	0.9042	0.9019	0.9016	0.9017	0.9020
KIMP%:	0.9021	0.9046	0.9040	0.9035	0.9024	0.9025	0.9027	0.9029

Table B.7. Average Coverage Probability for Model M3 T=50

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.9045	0.9210	0.9278	0.9309	0.9319	0.9311	0.9301	0.9296
PRRP:	0.9026	0.9119	0.9145	0.9124	0.9107	0.9083	0.9039	0.9010
KIM:	0.8852	0.8935	0.8971	0.8966	0.8962	0.8952	0.8928	0.8904
KIMP:	0.8819	0.8816	0.8769	0.8674	0.8575	0.8485	0.8403	0.8318
PRR%:	0.8970	0.8995	0.8961	0.8888	0.8800	0.8706	0.8606	0.8511
PRRP%:	0.9000	0.8992	0.8959	0.8883	0.8796	0.8696	0.8606	0.8507
KIM%:	0.8781	0.8735	0.8665	0.8555	0.8435	0.8317	0.8195	0.8072
KIMP%:	0.8801	0.8755	0.8665	0.8551	0.8397	0.8252	0.8114	0.7981

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9245	0.9307	0.9361	0.9401	0.9420	0.9420	0.9418	0.9409
PRRP:	0.9230	0.9225	0.9250	0.9247	0.9232	0.9205	0.9174	0.9141
KIM:	0.9115	0.9081	0.9089	0.9098	0.9096	0.9079	0.9063	0.9047
KIMP:	0.9087	0.8969	0.8907	0.8823	0.8734	0.8646	0.8574	0.8489
PRR%:	0.8732	0.9022	0.9086	0.9066	0.9009	0.8949	0.8875	0.8783
PRRP%:	0.8764	0.9033	0.9098	0.9080	0.9033	0.8964	0.8882	0.8801
KIM%:	0.8435	0.8697	0.8725	0.8679	0.8594	0.8487	0.8378	0.8258
KIMP%:	0.8446	0.8695	0.8717	0.8654	0.8565	0.8432	0.8309	0.8181

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9234	0.9240	0.9253	0.9267	0.9264	0.9266	0.9248	0.9234
PRRP:	0.9218	0.9162	0.9133	0.9107	0.9071	0.9041	0.9009	0.8968
KIM:	0.9158	0.9068	0.9016	0.8985	0.8958	0.8929	0.8901	0.8868
KIMP:	0.9069	0.8906	0.8780	0.8678	0.8570	0.8466	0.8385	0.8287
PRR%:	0.8538	0.8830	0.8916	0.8894	0.8839	0.8769	0.8678	0.8586
PRRP%:	0.8550	0.8833	0.8912	0.8883	0.8829	0.8754	0.8676	0.8588
KIM%:	0.8207	0.8463	0.8505	0.8445	0.8362	0.8255	0.8148	0.8034
KIMP%:	0.8206	0.8474	0.8527	0.8469	0.8378	0.8255	0.8131	0.8015

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8961	0.9091	0.9164	0.9206	0.9221	0.9219	0.9206	0.9201
PRRP:	0.8965	0.9020	0.9052	0.9051	0.9019	0.8985	0.8954	0.8924
KIM:	0.8807	0.8865	0.8874	0.8883	0.8884	0.8869	0.8855	0.8842
KIMP:	0.8794	0.8751	0.8684	0.8604	0.8503	0.8411	0.8330	0.8241
PRR%:	0.8991	0.8989	0.8949	0.8878	0.8789	0.8695	0.8602	0.8508
PRRP%:	0.9031	0.9020	0.8967	0.8891	0.8799	0.8711	0.8627	0.8544
KIM%:	0.8822	0.8750	0.8646	0.8535	0.8406	0.8279	0.8156	0.8031
KIMP%:	0.8848	0.8772	0.8678	0.8547	0.8404	0.8259	0.8130	0.7994

Table B.8. Average Coverage Probability for Model M3 T=100

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.9035	0.9177	0.9272	0.9334	0.9374	0.9383	0.9391	0.9407
PRRP:	0.9057	0.9166	0.9221	0.9255	0.9269	0.9269	0.9261	0.9240
KIM:	0.8949	0.9028	0.9081	0.9103	0.9120	0.9124	0.9129	0.9140
KIMP:	0.8935	0.8976	0.8965	0.8939	0.8905	0.8852	0.8799	0.8758
PRR%:	0.8999	0.9071	0.9105	0.9101	0.9097	0.9058	0.9023	0.8995
PRRP%:	0.9029	0.9091	0.9129	0.9123	0.9118	0.9084	0.9052	0.9017
KIM%:	0.8914	0.8907	0.8895	0.8853	0.8799	0.8750	0.8685	0.8629
KIMP%:	0.8922	0.8923	0.8900	0.8862	0.8806	0.8735	0.8666	0.8592

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9222	0.9252	0.9308	0.9352	0.9374	0.9393	0.9409	0.9409
PRRP:	0.9217	0.9216	0.9248	0.9270	0.9271	0.9275	0.9267	0.9249
KIM:	0.9135	0.9107	0.9127	0.9145	0.9155	0.9158	0.9165	0.9159
KIMP:	0.9104	0.9032	0.9001	0.8965	0.8911	0.8864	0.8818	0.8764
PRR%:	0.8472	0.8867	0.9019	0.9087	0.9098	0.9096	0.9068	0.9034
PRRP%:	0.8496	0.8889	0.9040	0.9099	0.9113	0.9104	0.9080	0.9054
KIM%:	0.8281	0.8650	0.8776	0.8802	0.8788	0.8753	0.8711	0.8655
KIMP%:	0.8280	0.8648	0.8770	0.8795	0.8781	0.8737	0.8685	0.8621

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9319	0.9303	0.9322	0.9346	0.9367	0.9375	0.9380	0.9384
PRRP:	0.9291	0.9242	0.9243	0.9249	0.9242	0.9234	0.9227	0.9212
KIM:	0.9253	0.9181	0.9159	0.9164	0.9167	0.9167	0.9161	0.9162
KIMP:	0.9200	0.9088	0.9011	0.8967	0.8921	0.8877	0.8821	0.8771
PRR%:	0.8317	0.8757	0.8965	0.9057	0.9094	0.9099	0.9086	0.9050
PRRP%:	0.8335	0.8774	0.8977	0.9063	0.9097	0.9109	0.9092	0.9079
KIM%:	0.8067	0.8487	0.8675	0.8734	0.8750	0.8728	0.8705	0.8663
KIMP%:	0.8056	0.8468	0.8648	0.8714	0.8744	0.8719	0.8682	0.8631

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.9012	0.9148	0.9215	0.9270	0.9290	0.9312	0.9325	0.9334
PRRP:	0.9015	0.9115	0.9168	0.9194	0.9214	0.9202	0.9191	0.9183
KIM:	0.8928	0.9010	0.9032	0.9075	0.9086	0.9084	0.9090	0.9109
KIMP:	0.8930	0.8964	0.8978	0.8963	0.8933	0.8878	0.8832	0.8796
PRR%:	0.9056	0.9112	0.9127	0.9136	0.9116	0.9091	0.9047	0.9021
PRRP%:	0.9074	0.9119	0.9144	0.9126	0.9117	0.9105	0.9059	0.9033
KIM%:	0.8959	0.8986	0.8945	0.8902	0.8858	0.8805	0.8756	0.8698
KIMP%:	0.8981	0.8974	0.8976	0.8948	0.8877	0.8804	0.8742	0.8683

Table B.9. Average Coverage Probability for Model M3 T=200

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.9024	0.9133	0.9207	0.9268	0.9307	0.9338	0.9373	0.9386
PRRP:	0.9037	0.9124	0.9196	0.9255	0.9289	0.9309	0.9327	0.9334
KIM:	0.9003	0.9078	0.9127	0.9169	0.9194	0.9214	0.9228	0.9241
KIMP:	0.8995	0.9049	0.9068	0.9083	0.9088	0.9077	0.9060	0.9042
PRR%:	0.9008	0.9077	0.9117	0.9151	0.9155	0.9152	0.9152	0.9143
PRRP%:	0.9023	0.9089	0.9135	0.9175	0.9192	0.9186	0.9190	0.9185
KIM%:	0.8992	0.9034	0.9060	0.9072	0.9064	0.9048	0.9026	0.8999
KIMP%:	0.8984	0.9015	0.9033	0.9033	0.9023	0.9002	0.8976	0.8949

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9190	0.9203	0.9266	0.9304	0.9344	0.9373	0.9395	0.9411
PRRP:	0.9194	0.9204	0.9253	0.9287	0.9310	0.9328	0.9346	0.9352
KIM:	0.9096	0.9113	0.9146	0.9172	0.9200	0.9218	0.9232	0.9247
KIMP:	0.9113	0.9090	0.9093	0.9097	0.9096	0.9083	0.9070	0.9050
PRR%:	0.8337	0.8790	0.8981	0.9064	0.9114	0.9137	0.9147	0.9149
PRRP%:	0.8359	0.8818	0.9005	0.9095	0.9147	0.9169	0.9187	0.9186
KIM%:	0.8229	0.8672	0.8851	0.8919	0.8957	0.8966	0.8971	0.8961
KIMP%:	0.8235	0.8672	0.8849	0.8912	0.8938	0.8947	0.8938	0.8920

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9313	0.9272	0.9296	0.9334	0.9362	0.9374	0.9391	0.9401
PRRP:	0.9311	0.9266	0.9282	0.9308	0.9324	0.9340	0.9343	0.9346
KIM:	0.9220	0.9168	0.9180	0.9196	0.9217	0.9229	0.9239	0.9245
KIMP:	0.9235	0.9141	0.9124	0.9124	0.9107	0.9084	0.9061	0.9041
PRR%:	0.8079	0.8574	0.8824	0.8968	0.9055	0.9100	0.9124	0.9132
PRRP%:	0.8107	0.8603	0.8851	0.8998	0.9081	0.9127	0.9154	0.9160
KIM%:	0.7936	0.8426	0.8669	0.8804	0.8876	0.8913	0.8918	0.8924
KIMP%:	0.7941	0.8427	0.8670	0.8790	0.8857	0.8883	0.8892	0.8886

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.9022	0.9126	0.9203	0.9251	0.9290	0.9325	0.9348	0.9361
PRRP:	0.9024	0.9120	0.9189	0.9235	0.9258	0.9279	0.9295	0.9308
KIM:	0.8989	0.9082	0.9132	0.9170	0.9191	0.9204	0.9223	0.9232
KIMP:	0.8990	0.9053	0.9087	0.9093	0.9084	0.9066	0.9055	0.9040
PRR%:	0.9048	0.9135	0.9180	0.9199	0.9205	0.9204	0.9198	0.9185
PRRP%:	0.9060	0.9137	0.9188	0.9212	0.9220	0.9221	0.9221	0.9216
KIM%:	0.9007	0.9079	0.9105	0.9106	0.9092	0.9082	0.9057	0.9034
KIMP%:	0.9007	0.9067	0.9089	0.9084	0.9069	0.9047	0.9021	0.8999

Table B.10. Average Coverage Probability for Model M4 T=50

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.9033	0.9196	0.9275	0.9292	0.9294	0.9291	0.9274	0.9253
PRRP:	0.8996	0.9078	0.9086	0.9055	0.9016	0.8969	0.8919	0.8878
KIM:	0.8842	0.8931	0.8967	0.8972	0.8972	0.8961	0.8941	0.8928
KIMP:	0.8809	0.8776	0.8709	0.8609	0.8494	0.8395	0.8289	0.8197
PRR%:	0.8960	0.8994	0.8979	0.8919	0.8837	0.8756	0.8663	0.8571
PRRP%:	0.8969	0.8989	0.8945	0.8872	0.8786	0.8694	0.8594	0.8505
KIM%:	0.8737	0.8679	0.8598	0.8483	0.8356	0.8231	0.8106	0.7984
KIMP%:	0.8762	0.8699	0.8603	0.8481	0.8343	0.8204	0.8073	0.7922

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9189	0.9227	0.9281	0.9290	0.9303	0.9300	0.9278	0.9259
PRRP:	0.9121	0.9109	0.9104	0.9077	0.9042	0.8996	0.8953	0.8909
KIM:	0.9026	0.8979	0.8968	0.8960	0.8944	0.8914	0.8886	0.8863
KIMP:	0.8938	0.8805	0.8716	0.8609	0.8501	0.8397	0.8293	0.8197
PRR%:	0.8706	0.8933	0.8978	0.8941	0.8875	0.8780	0.8701	0.8603
PRRP%:	0.8708	0.8929	0.8956	0.8909	0.8835	0.8735	0.8647	0.8553
KIM%:	0.8405	0.8586	0.8582	0.8504	0.8399	0.8278	0.8165	0.8060
KIMP%:	0.8422	0.8629	0.8637	0.8556	0.8446	0.8319	0.8192	0.8066

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9253	0.9251	0.9278	0.9291	0.9293	0.9288	0.9281	0.9265
PRRP:	0.9184	0.9125	0.9095	0.9066	0.9038	0.8991	0.8959	0.8913
KIM:	0.9158	0.9071	0.9038	0.8996	0.8962	0.8917	0.8885	0.8848
KIMP:	0.9007	0.8857	0.8748	0.8636	0.8518	0.8404	0.8311	0.8204
PRR%:	0.8630	0.8932	0.9003	0.8972	0.8911	0.8815	0.8733	0.8651
PRRP%:	0.8639	0.8923	0.8986	0.8954	0.8873	0.8783	0.8690	0.8595
KIM%:	0.8259	0.8512	0.8564	0.8521	0.8446	0.8341	0.8236	0.8121
KIMP%:	0.8303	0.8590	0.8655	0.8603	0.8510	0.8390	0.8284	0.8152

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.9010	0.9130	0.9203	0.9226	0.9230	0.9229	0.9219	0.9202
PRRP:	0.8986	0.9045	0.9047	0.9018	0.8982	0.8932	0.8898	0.8842
KIM:	0.8821	0.8853	0.8865	0.8847	0.8833	0.8819	0.8801	0.8767
KIMP:	0.8801	0.8769	0.8675	0.8570	0.8455	0.8337	0.8235	0.8125
PRR%:	0.9011	0.9017	0.8984	0.8915	0.8822	0.8731	0.8641	0.8548
PRRP%:	0.9030	0.9005	0.8959	0.8877	0.8784	0.8693	0.8592	0.8499
KIM%:	0.8822	0.8732	0.8640	0.8520	0.8400	0.8280	0.8163	0.8052
KIMP%:	0.8834	0.8770	0.8690	0.8567	0.8446	0.8314	0.8182	0.8058

Table B.11. Average Coverage Probability for Model M4 T=100

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.9066	0.9233	0.9337	0.9399	0.9434	0.9467	0.9481	0.9490
PRRP:	0.9051	0.9178	0.9238	0.9269	0.9276	0.9268	0.9262	0.9247
KIM:	0.8923	0.9028	0.9071	0.9086	0.9102	0.9107	0.9102	0.9106
KIMP:	0.8904	0.8950	0.8948	0.8917	0.8879	0.8819	0.8764	0.8707
PRR%:	0.9020	0.9116	0.9158	0.9172	0.9173	0.9150	0.9131	0.9108
PRRP%:	0.9017	0.9102	0.9123	0.9130	0.9121	0.9095	0.9065	0.9028
KIM%:	0.8877	0.8921	0.8896	0.8858	0.8808	0.8757	0.8690	0.8632
KIMP%:	0.8888	0.8923	0.8913	0.8888	0.8836	0.8778	0.8716	0.8649

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9263	0.9289	0.9343	0.9384	0.9409	0.9426	0.9430	0.9435
PRRP:	0.9200	0.9207	0.9227	0.9239	0.9239	0.9226	0.9210	0.9195
KIM:	0.9151	0.9109	0.9112	0.9121	0.9125	0.9122	0.9105	0.9094
KIMP:	0.9079	0.9005	0.8961	0.8919	0.8872	0.8814	0.8754	0.8689
PRR%:	0.8582	0.8972	0.9120	0.9164	0.9168	0.9150	0.9124	0.9085
PRRP%:	0.8580	0.8974	0.9111	0.9143	0.9149	0.9114	0.9091	0.9050
KIM%:	0.8319	0.8693	0.8797	0.8817	0.8784	0.8748	0.8695	0.8643
KIMP%:	0.8342	0.8721	0.8847	0.8867	0.8859	0.8812	0.8764	0.8700

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9319	0.9298	0.9336	0.9368	0.9393	0.9405	0.9413	0.9417
PRRP:	0.9249	0.9205	0.9202	0.9210	0.9214	0.9206	0.9181	0.9161
KIM:	0.9262	0.9181	0.9170	0.9154	0.9145	0.9136	0.9125	0.9116
KIMP:	0.9150	0.9026	0.8967	0.8918	0.8856	0.8801	0.8739	0.8672
PRR%:	0.8348	0.8774	0.8962	0.9057	0.9102	0.9107	0.9093	0.9071
PRRP%:	0.8354	0.8769	0.8951	0.9038	0.9068	0.9063	0.9049	0.9027
KIM%:	0.8061	0.8479	0.8643	0.8702	0.8709	0.8690	0.8659	0.8619
KIMP%:	0.8089	0.8494	0.8667	0.8745	0.8750	0.8734	0.8693	0.8638

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.9086	0.9190	0.9282	0.9316	0.9364	0.9362	0.9359	0.9363
PRRP:	0.9086	0.9159	0.9190	0.9210	0.9211	0.9198	0.9177	0.9164
KIM:	0.8998	0.9016	0.9038	0.9029	0.9039	0.9020	0.8998	0.8982
KIMP:	0.8984	0.8965	0.8915	0.8870	0.8813	0.8764	0.8682	0.8646
PRR%:	0.9122	0.9144	0.9150	0.9135	0.9119	0.9076	0.9043	0.9002
PRRP%:	0.9107	0.9128	0.9132	0.9104	0.9071	0.9015	0.8993	0.8945
KIM%:	0.9006	0.8984	0.8931	0.8851	0.8796	0.8720	0.8673	0.8627
KIMP%:	0.9009	0.8979	0.8938	0.8893	0.8842	0.8754	0.8685	0.8627

Table B.12. Average Coverage Probability for Model M4 T=200

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.9062	0.9181	0.9263	0.9334	0.9379	0.9416	0.9439	0.9453
PRRP:	0.9043	0.9151	0.9219	0.9265	0.9290	0.9305	0.9311	0.9302
KIM:	0.8981	0.9053	0.9094	0.9128	0.9149	0.9161	0.9170	0.9171
KIMP:	0.8987	0.9027	0.9031	0.9046	0.9021	0.9000	0.8965	0.8932
PRR%:	0.9031	0.9123	0.9170	0.9213	0.9229	0.9238	0.9241	0.9229
PRRP%:	0.9026	0.9117	0.9163	0.9197	0.9209	0.9207	0.9205	0.9195
KIM%:	0.8960	0.9010	0.9024	0.9025	0.9013	0.9002	0.8976	0.8952
KIMP%:	0.8974	0.9019	0.9033	0.9048	0.9031	0.9015	0.8986	0.8957

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9197	0.9226	0.9290	0.9354	0.9390	0.9416	0.9440	0.9453
PRRP:	0.9156	0.9175	0.9219	0.9260	0.9281	0.9297	0.9304	0.9302
KIM:	0.9096	0.9094	0.9120	0.9134	0.9151	0.9156	0.9162	0.9166
KIMP:	0.9034	0.9027	0.9022	0.9011	0.8990	0.8968	0.8934	0.8904
PRR%:	0.8406	0.8864	0.9052	0.9159	0.9211	0.9227	0.9235	0.9236
PRRP%:	0.8416	0.8868	0.9056	0.9152	0.9197	0.9215	0.9217	0.9210
KIM%:	0.8248	0.8687	0.8851	0.8919	0.8945	0.8947	0.8929	0.8917
KIMP%:	0.8252	0.8694	0.8863	0.8948	0.8983	0.8988	0.8975	0.8968

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9311	0.9291	0.9329	0.9369	0.9398	0.9421	0.9436	0.9451
PRRP:	0.9265	0.9217	0.9238	0.9258	0.9283	0.9295	0.9299	0.9302
KIM:	0.9275	0.9177	0.9167	0.9173	0.9166	0.9170	0.9171	0.9169
KIMP:	0.9169	0.9062	0.9048	0.9036	0.9014	0.8991	0.8959	0.8931
PRR%:	0.8216	0.8711	0.8953	0.9092	0.9159	0.9203	0.9219	0.9229
PRRP%:	0.8233	0.8722	0.8958	0.9089	0.9159	0.9191	0.9202	0.9209
KIM%:	0.7990	0.8463	0.8688	0.8806	0.8856	0.8879	0.8894	0.8886
KIMP%:	0.8006	0.8482	0.8711	0.8834	0.8889	0.8917	0.8922	0.8923

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.9013	0.9142	0.9234	0.9311	0.9353	0.9391	0.9409	0.9425
PRRP:	0.9020	0.9128	0.9205	0.9258	0.9278	0.9299	0.9306	0.9304
KIM:	0.8951	0.9030	0.9074	0.9116	0.9123	0.9133	0.9135	0.9134
KIMP:	0.8954	0.9013	0.9028	0.9050	0.9040	0.9020	0.8992	0.8958
PRR%:	0.9039	0.9132	0.9191	0.9235	0.9252	0.9263	0.9260	0.9250
PRRP%:	0.9046	0.9134	0.9192	0.9230	0.9238	0.9245	0.9237	0.9220
KIM%:	0.8964	0.9022	0.9045	0.9053	0.9045	0.9019	0.8995	0.8970
KIMP%:	0.8976	0.9039	0.9065	0.9084	0.9076	0.9054	0.9031	0.9005

Table B.13. Average Coverage Probability for Model M5 T=50

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.9214	0.9214	0.9176	0.9136	0.9105	0.9061	0.9010	0.8982
PRRP:	0.9174	0.9078	0.8973	0.8877	0.8793	0.8708	0.8615	0.8554
KIM:	0.8835	0.8694	0.8564	0.8467	0.8394	0.8330	0.8261	0.8214
KIMP:	0.8778	0.8520	0.8263	0.8060	0.7887	0.7750	0.7613	0.7494
PRR%:	0.9083	0.8951	0.8765	0.8574	0.8378	0.8163	0.7956	0.7770
PRRP%:	0.9100	0.8928	0.8714	0.8484	0.8268	0.8052	0.7821	0.7627
KIM%:	0.8725	0.8465	0.8196	0.7938	0.7721	0.7506	0.7283	0.7081
KIMP%:	0.8758	0.8492	0.8197	0.7927	0.7659	0.7419	0.7159	0.6941

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9278	0.9211	0.9170	0.9129	0.9077	0.9041	0.9004	0.8977
PRRP:	0.9205	0.9075	0.8959	0.8864	0.8779	0.8695	0.8618	0.8543
KIM:	0.9062	0.8799	0.8647	0.8546	0.8452	0.8391	0.8342	0.8302
KIMP:	0.8933	0.8561	0.8338	0.8139	0.7976	0.7846	0.7722	0.7615
PRR%:	0.8878	0.8944	0.8836	0.8682	0.8489	0.8296	0.8124	0.7940
PRRP%:	0.8879	0.8915	0.8786	0.8607	0.8393	0.8196	0.7995	0.7807
KIM%:	0.8438	0.8384	0.8225	0.8024	0.7826	0.7622	0.7423	0.7253

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9343	0.9224	0.9152	0.9102	0.9060	0.9015	0.8965	0.8931
PRRP:	0.9271	0.9064	0.8936	0.8831	0.8721	0.8631	0.8549	0.8487
KIM:	0.9180	0.8896	0.8696	0.8582	0.8474	0.8400	0.8345	0.8292
KIMP:	0.9069	0.8657	0.8345	0.8122	0.7966	0.7806	0.7665	0.7550
PRR%:	0.8779	0.8880	0.8805	0.8662	0.8495	0.8304	0.8120	0.7928
PRRP%:	0.8784	0.8850	0.8755	0.8597	0.8414	0.8211	0.8016	0.7821
KIM%:	0.8380	0.8401	0.8262	0.8074	0.7872	0.7689	0.7482	0.7300
KIMP%:	0.8409	0.8444	0.8314	0.8115	0.7908	0.7685	0.7462	0.7257

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.9127	0.9109	0.9067	0.9025	0.8983	0.8937	0.8898	0.8866
PRRP:	0.9107	0.9001	0.8883	0.8781	0.8689	0.8604	0.8517	0.8441
KIM:	0.8810	0.8643	0.8523	0.8438	0.8363	0.8309	0.8264	0.8241
KIMP:	0.8776	0.8501	0.8252	0.8070	0.7910	0.7760	0.7644	0.7532
PRR%:	0.9111	0.8974	0.8791	0.8603	0.8404	0.8212	0.8020	0.7850
PRRP%:	0.9128	0.8970	0.8760	0.8547	0.8335	0.8128	0.7922	0.7732
KIM%:	0.8766	0.8486	0.8201	0.7949	0.7727	0.7502	0.7310	0.7124
KIMP%:	0.8827	0.8536	0.8235	0.7968	0.7723	0.7465	0.7249	0.7022

Table B.14. Average Coverage Probability for Model M5 T=100

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.9179	0.9251	0.9261	0.9249	0.9235	0.9214	0.9188	0.9169
PRRP:	0.9156	0.9146	0.9100	0.9055	0.9002	0.8947	0.8889	0.8853
KIM:	0.8919	0.8883	0.8815	0.8751	0.8701	0.8645	0.8606	0.8555
KIMP:	0.8892	0.8773	0.8617	0.8480	0.8358	0.8242	0.8146	0.8047
PRR%:	0.9130	0.9141	0.9081	0.9002	0.8909	0.8823	0.8731	0.8638
PRRP%:	0.9147	0.9128	0.9052	0.8966	0.8858	0.8749	0.8635	0.8536
KIM%:	0.8873	0.8768	0.8620	0.8459	0.8334	0.8205	0.8069	0.7936
KIMP%:	0.8896	0.8792	0.8645	0.8483	0.8334	0.8173	0.8030	0.7876

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9321	0.9275	0.9261	0.9242	0.9236	0.9208	0.9194	0.9168
PRRP:	0.9247	0.9163	0.9106	0.9057	0.9012	0.8958	0.8914	0.8868
KIM:	0.9128	0.8950	0.8842	0.8761	0.8696	0.8626	0.8577	0.8524
KIMP:	0.9034	0.8805	0.8618	0.8479	0.8344	0.8221	0.8119	0.8018
PRR%:	0.8711	0.8951	0.8994	0.8967	0.8904	0.8821	0.8742	0.8643
PRRP%:	0.8711	0.8938	0.8964	0.8916	0.8844	0.8759	0.8651	0.8537
KIM%:	0.8373	0.8565	0.8546	0.8472	0.8373	0.8250	0.8145	0.8027
KIMP%:	0.8384	0.8569	0.8555	0.8481	0.8365	0.8238	0.8109	0.7971

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9397	0.9314	0.9270	0.9249	0.9221	0.9194	0.9163	0.9150
PRRP:	0.9321	0.9180	0.9098	0.9038	0.8975	0.8926	0.8873	0.8817
KIM:	0.9325	0.9091	0.8955	0.8860	0.8777	0.8717	0.8662	0.8610
KIMP:	0.9181	0.8902	0.8699	0.8549	0.8406	0.8301	0.8205	0.8109
PRR%:	0.8628	0.8873	0.8974	0.8978	0.8929	0.8873	0.8795	0.8706
PRRP%:	0.8619	0.8854	0.8939	0.8933	0.8884	0.8813	0.8727	0.8627
KIM%:	0.8220	0.8406	0.8459	0.8422	0.8340	0.8257	0.8148	0.8049
KIMP%:	0.8232	0.8420	0.8477	0.8439	0.8360	0.8275	0.8156	0.8032

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.9116	0.9191	0.9206	0.9197	0.9206	0.9186	0.9174	0.9160
PRRP:	0.9104	0.9122	0.9088	0.9044	0.9007	0.8965	0.8921	0.8879
KIM:	0.8888	0.8827	0.8754	0.8699	0.8637	0.8583	0.8554	0.8522
KIMP:	0.8873	0.8778	0.8616	0.8485	0.8358	0.8254	0.8156	0.8051
PRR%:	0.9114	0.9142	0.9092	0.9032	0.8959	0.8858	0.8793	0.8686
PRRP%:	0.9123	0.9117	0.9068	0.8984	0.8890	0.8811	0.8681	0.8584
KIM%:	0.8910	0.8786	0.8651	0.8527	0.8416	0.8273	0.8153	0.8056
KIMP%:	0.8913	0.8832	0.8700	0.8565	0.8420	0.8263	0.8140	0.7993

Table B.15. Average Coverage Probability for Model M5 T=200

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.9142	0.9210	0.9249	0.9263	0.9266	0.9263	0.9262	0.9259
PRRP:	0.9130	0.9161	0.9153	0.9148	0.9128	0.9105	0.9080	0.9054
KIM:	0.8978	0.8965	0.8911	0.8898	0.8849	0.8814	0.8775	0.8733
KIMP:	0.8965	0.8904	0.8821	0.8739	0.8648	0.8588	0.8513	0.8438
PRR%:	0.9109	0.9145	0.9145	0.9129	0.9098	0.9060	0.9020	0.8985
PRRP%:	0.9113	0.9131	0.9112	0.9091	0.9055	0.9005	0.8958	0.8905
KIM%:	0.8944	0.8896	0.8817	0.8760	0.8684	0.8601	0.8533	0.8458
KIMP%:	0.8955	0.8906	0.8834	0.8755	0.8679	0.8599	0.8509	0.8435

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9314	0.9285	0.9284	0.9287	0.9279	0.9284	0.9281	0.9272
PRRP:	0.9259	0.9201	0.9181	0.9147	0.9130	0.9102	0.9078	0.9057
KIM:	0.9158	0.9041	0.8963	0.8914	0.8872	0.8826	0.8790	0.8759
KIMP:	0.9110	0.8955	0.8856	0.8760	0.8677	0.8595	0.8538	0.8455
PRR%:	0.8569	0.8902	0.9035	0.9076	0.9081	0.9068	0.9044	0.9011
PRRP%:	0.8574	0.8902	0.9024	0.9055	0.9056	0.9030	0.8995	0.8956
KIM%:	0.8275	0.8560	0.8645	0.8643	0.8609	0.8562	0.8513	0.8453
KIMP%:	0.8304	0.8589	0.8687	0.8689	0.8661	0.8609	0.8554	0.8486

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9406	0.9310	0.9282	0.9273	0.9265	0.9263	0.9253	0.9254
PRRP:	0.9314	0.9190	0.9161	0.9138	0.9118	0.9096	0.9077	0.9042
KIM:	0.9335	0.9122	0.9028	0.8955	0.8904	0.8856	0.8822	0.8792
KIMP:	0.9217	0.8975	0.8852	0.8756	0.8675	0.8588	0.8515	0.8448
PRR%:	0.8358	0.8682	0.8853	0.8953	0.8990	0.8996	0.8999	0.8975
PRRP%:	0.8355	0.8668	0.8833	0.8922	0.8957	0.8958	0.8948	0.8923
KIM%:	0.8036	0.8316	0.8451	0.8514	0.8533	0.8515	0.8484	0.8445
KIMP%:	0.8043	0.8321	0.8465	0.8540	0.8551	0.8538	0.8504	0.8456

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.9119	0.9176	0.9206	0.9231	0.9236	0.9239	0.9248	0.9249
PRRP:	0.9104	0.9152	0.9152	0.9144	0.9132	0.9114	0.9098	0.9077
KIM:	0.8980	0.8962	0.8916	0.8880	0.8832	0.8799	0.8759	0.8722
KIMP:	0.8972	0.8914	0.8838	0.8755	0.8678	0.8603	0.8532	0.8470
PRR%:	0.9123	0.9158	0.9155	0.9153	0.9124	0.9093	0.9068	0.9034
PRRP%:	0.9126	0.9158	0.9139	0.9128	0.9086	0.9058	0.9008	0.8967
KIM%:	0.8993	0.8949	0.8878	0.8811	0.8742	0.8674	0.8611	0.8540
KIMP%:	0.9000	0.8955	0.8889	0.8824	0.8755	0.8682	0.8609	0.8545

Table B.16. Variance and Lengths of Intervals for Model M1 T=50

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	2.239	2.334	2.118	2.270	2.195	2.339	2.437	2.396
Var	3	2.079	1.948	1.956	2.063	2.519	2.295	2.580	2.599
	8	2.261	2.160	2.251	2.274	2.632	2.516	2.861	2.717
	1	4.038	4.044	4.053	4.031	4.007	4.015	4.027	4.006
Len1	3	4.715	4.788	4.720	4.747	4.562	4.573	4.582	4.572
	8	4.860	4.969	4.853	4.907	4.611	4.624	4.626	4.629
	1	4.028	4.034	4.034	4.032	3.995	4.007	4.005	4.006
Len2	3	5.463	5.505	5.466	5.390	5.187	5.206	5.191	5.199
	8	5.783	5.843	5.749	5.609	5.307	5.309	5.292	5.291
Chi	1	3.537	3.773	3.636	3.455	2.389	2.246	2.747	2.674
Var	3	2.527	2.646	2.446	2.552	3.169	2.996	3.604	3.508
	8	2.814	2.841	2.629	2.727	3.477	3.276	4.006	3.811
	1	3.971	4.017	3.973	3.994	4.072	4.101	3.968	3.987
Len1	3	4.796	4.866	4.777	4.830	4.735	4.742	4.620	4.643
	8	4.902	5.004	4.901	4.945	4.758	4.754	4.653	4.654
	1	3.985	3.998	4.017	4.041	4.068	4.075	4.018	4.022
Len2	3	5.429	5.436	5.421	5.341	5.247	5.237	5.154	5.160
	8	5.708	5.791	5.702	5.561	5.338	5.358	5.256	5.253
Exp	1	3.911	4.141	3.886	2.960	2.657	2.588	3.245	3.208
Var	3	2.872	2.894	2.837	2.679	3.545	3.322	4.122	4.022
	8	2.748	2.714	2.563	2.760	3.715	3.466	4.514	4.384
	1	3.978	4.012	3.984	3.988	4.171	4.189	3.996	4.005
Len1	3	4.883	4.932	4.888	4.964	4.894	4.886	4.732	4.759
	8	5.012	5.096	4.983	5.041	4.923	4.915	4.737	4.748
	1	4.008	4.017	4.041	4.064	4.183	4.190	4.078	4.095
Len2	3	5.492	5.538	5.486	5.432	5.399	5.421	5.261	5.269
	8	5.802	5.878	5.771	5.668	5.500	5.534	5.339	5.366
t-dist	1	2.317	2.371	2.220	2.258	1.946	1.995	2.151	2.199
Var	3	2.181	2.260	2.145	2.240	2.608	2.621	2.789	2.708
	8	2.257	2.268	2.229	2.363	2.715	2.560	2.870	2.782
	1	4.250	4.270	4.230	4.294	4.304	4.322	4.211	4.265
Len1	3	4.897	4.977	4.910	4.942	4.839	4.851	4.744	4.749
	8	5.049	5.115	5.022	5.105	4.876	4.871	4.761	4.806
	1	4.147	4.158	4.171	4.192	4.178	4.197	4.140	4.170
Len2	3	5.619	5.638	5.596	5.538	5.402	5.408	5.309	5.343
	8	5.882	5.941	5.864	5.751	5.483	5.476	5.372	5.412

Table B.17. Variance and Lengths of Intervals for Model M2 T=50

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	2.089	2.216	2.253	2.251	1.938	1.959	2.341	2.311
Var	3	2.472	2.754	2.289	2.667	3.073	3.005	3.528	3.784
	8	3.312	3.671	3.149	3.652	4.100	4.081	4.660	4.373
	1	4.083	4.103	4.080	4.068	4.054	4.082	4.035	4.042
Len1	3	4.921	4.911	4.905	4.764	4.618	4.636	4.586	4.610
	8	5.230	5.205	5.223	4.920	4.662	4.680	4.625	4.637
	1	4.021	4.026	4.027	4.026	3.994	4.004	3.992	3.999
Len2	3	6.039	6.023	6.043	5.832	5.671	5.710	5.662	5.647
	8	7.151	7.070	7.080	6.511	6.107	6.150	6.036	6.031
Chi	1	3.402	3.295	4.059	3.837	2.369	2.317	2.796	2.718
Var	3	2.847	3.008	3.650	3.931	4.516	4.258	4.971	5.033
	8	3.074	3.332	3.317	3.803	4.827	4.560	5.789	5.483
	1	4.066	4.063	4.075	4.082	4.155	4.175	4.060	4.074
Len1	3	4.892	4.914	4.915	4.772	4.725	4.770	4.598	4.619
	8	5.250	5.238	5.228	4.960	4.817	4.847	4.637	4.664
	1	4.018	4.025	4.033	4.035	4.103	4.114	4.023	4.031
Len2	3	6.062	6.037	6.065	5.877	5.782	5.826	5.687	5.705
	8	7.180	7.127	7.105	6.534	6.251	6.299	6.085	6.073
Exp	1	2.949	2.649	3.003	2.652	2.427	2.287	3.003	3.021
Var	3	2.507	2.646	2.623	2.797	3.627	3.396	4.508	4.520
	8	2.712	3.003	2.603	3.216	4.579	4.407	5.536	5.378
	1	3.973	3.986	3.967	3.964	4.161	4.187	3.969	3.993
Len1	3	4.810	4.822	4.826	4.699	4.749	4.792	4.524	4.553
	8	5.134	5.101	5.140	4.885	4.819	4.837	4.570	4.592
	1	4.054	4.078	4.048	4.082	4.214	4.240	4.071	4.102
Len2	3	5.981	5.989	5.968	5.800	5.803	5.852	5.627	5.659
	8	7.008	6.977	6.973	6.428	6.200	6.266	5.995	5.993
t-dist	1	2.201	2.271	2.372	2.296	2.102	2.032	2.160	2.271
Var	3	2.884	2.798	2.824	3.033	3.381	3.209	3.488	3.704
	8	3.439	3.787	3.398	3.835	4.116	3.896	4.184	4.128
	1	4.200	4.209	4.249	4.240	4.237	4.254	4.201	4.216
Len1	3	5.009	5.014	4.997	4.877	4.766	4.794	4.675	4.694
	8	5.268	5.244	5.255	5.009	4.760	4.783	4.662	4.712
	1	4.255	4.242	4.223	4.274	4.278	4.280	4.187	4.248
Len2	3	6.157	6.159	6.149	5.982	5.856	5.908	5.768	5.809
	8	7.182	7.138	7.163	6.592	6.228	6.279	6.119	6.114

Table B.18. Variance and Lengths of Intervals for Model M3 T=50

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	2.353	2.485	2.341	2.740	2.243	2.123	2.635	2.456
Var	3	4.136	5.714	3.643	5.527	5.624	5.097	5.988	4.930
	8	10.599	15.660	10.302	16.794	18.667	17.759	17.573	16.748
	1	4.290	4.365	3.990	3.997	4.295	4.429	3.931	3.959
Len1	3	7.929	7.872	6.909	6.616	7.832	8.232	6.424	6.511
	8	13.653	12.520	11.391	9.836	12.865	13.799	9.121	9.343
	1	4.232	4.253	4.134	4.134	4.204	4.250	4.106	4.087
Len2	3	7.026	7.041	6.465	6.317	6.891	7.130	6.168	6.172
	8	12.927	12.052	10.987	9.805	12.203	12.897	9.181	9.318
Chi	1	2.431	2.453	2.764	3.155	3.181	3.319	3.060	3.370
Var	3	3.020	4.138	3.372	4.656	4.963	4.496	6.205	6.043
	8	7.539	11.653	7.398	13.332	13.806	12.947	15.668	14.559
	1	4.428	4.488	4.082	4.091	4.527	4.657	4.040	4.066
Len1	3	8.056	7.991	7.023	6.727	8.096	8.484	6.576	6.664
	8	13.985	12.751	11.597	9.951	13.183	14.093	9.352	9.605
	1	4.229	4.238	4.102	4.146	4.276	4.320	4.105	4.129
Len2	3	7.128	7.115	6.550	6.426	7.093	7.308	6.301	6.336
	8	13.300	12.317	11.254	9.978	12.603	13.268	9.469	9.618
Exp	1	4.020	4.629	3.859	5.642	3.756	4.106	3.543	3.827
Var	3	4.985	7.292	5.296	8.793	6.564	6.755	7.193	7.066
	8	11.464	16.359	10.916	17.644	17.353	16.958	18.988	17.153
	1	4.288	4.370	3.955	3.981	4.453	4.598	3.926	3.978
Len1	3	7.682	7.639	6.746	6.463	7.861	8.236	6.340	6.430
	8	13.158	12.068	11.084	9.594	12.662	13.535	9.006	9.242
	1	4.162	4.218	4.052	4.071	4.297	4.368	4.098	4.105
Len2	3	6.941	6.925	6.430	6.311	6.990	7.191	6.210	6.249
	8	12.557	11.746	10.868	9.689	12.056	12.737	9.164	9.334
t-dist	1	1.971	2.242	2.348	2.248	1.822	1.779	2.446	2.064
Var	3	4.236	5.494	3.746	5.278	4.950	4.696	6.751	5.172
	8	11.327	16.030	9.203	15.408	18.034	16.986	18.705	16.580
	1	4.416	4.498	4.139	4.167	4.468	4.609	4.075	4.124
Len1	3	7.834	7.776	6.909	6.651	7.867	8.260	6.445	6.566
	8	13.307	12.242	11.235	9.711	12.722	13.656	9.035	9.290
	1	4.276	4.345	4.229	4.208	4.297	4.375	4.208	4.176
Len2	3	7.015	7.017	6.504	6.383	6.954	7.176	6.237	6.278
	8	12.815	11.982	11.001	9.843	12.259	12.976	9.216	9.417

Table B.19. Variance and Lengths of Intervals for Model M4 T=50

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	2.546	2.709	2.431	2.659	2.591	2.431	3.407	2.724
Var	3	4.280	5.763	3.491	6.134	6.067	5.274	8.309	6.280
	8	11.451	15.842	7.818	16.536	18.922	16.735	20.254	17.328
	1	4.319	4.389	3.992	4.002	4.319	4.438	3.912	3.940
Len1	3	8.181	7.959	6.969	6.673	8.140	8.425	6.488	6.605
	8	14.420	12.802	11.778	10.202	14.139	14.829	9.581	9.984
	1	4.229	4.240	4.120	4.127	4.202	4.233	4.092	4.079
Len2	3	7.213	7.125	6.544	6.411	7.104	7.273	6.232	6.286
	8	13.633	12.356	11.406	10.178	13.294	13.781	9.571	9.914
Chi	1	3.220	4.075	3.428	4.780	3.731	4.173	3.122	3.467
Var	3	4.786	6.457	4.068	7.195	6.650	6.396	7.679	6.554
	8	11.324	15.906	8.416	17.570	19.517	17.833	19.314	17.232
	1	4.369	4.437	3.973	4.000	4.486	4.611	3.930	3.975
Len1	3	8.264	8.072	6.890	6.686	8.496	8.776	6.492	6.653
	8	14.644	13.113	11.750	10.452	15.176	15.885	9.770	10.256
	1	4.251	4.269	4.105	4.157	4.322	4.338	4.110	4.130
Len2	3	7.194	7.113	6.428	6.394	7.234	7.375	6.196	6.285
	8	13.641	12.461	11.221	10.235	13.910	14.384	9.622	10.010
Exp	1	4.198	5.777	3.555	6.376	4.282	4.643	3.101	3.277
Var	3	5.050	7.338	4.625	9.135	6.508	6.632	7.278	6.529
	8	11.923	17.242	9.677	18.905	18.883	17.578	19.643	18.191
	1	4.488	4.539	4.004	4.049	4.687	4.808	3.997	4.058
Len1	3	8.348	8.211	6.922	6.766	8.778	9.131	6.584	6.785
	8	14.771	13.234	11.762	10.505	15.732	16.545	9.920	10.425
	1	4.307	4.317	4.139	4.168	4.439	4.459	4.160	4.165
Len2	3	7.278	7.191	6.485	6.454	7.455	7.606	6.300	6.415
	8	13.759	12.557	11.223	10.381	14.354	14.910	9.761	10.201
t-dist	1	2.144	2.274	2.107	2.304	2.078	1.955	2.535	2.536
Var	3	4.288	5.441	3.583	5.343	5.796	5.417	6.809	6.081
	8	11.953	16.278	9.693	16.492	19.550	18.287	19.750	17.441
	1	4.479	4.509	4.113	4.146	4.550	4.650	4.059	4.116
Len1	3	8.225	8.016	6.941	6.700	8.423	8.732	6.527	6.684
	8	14.400	12.855	11.615	10.197	14.870	15.620	9.674	10.080
	1	4.406	4.416	4.265	4.264	4.433	4.461	4.239	4.224
Len2	3	7.228	7.119	6.538	6.431	7.240	7.371	6.277	6.333
	8	13.495	12.261	11.197	10.141	13.612	14.095	9.594	9.885

Table B.20. Variance and Lengths of Intervals for Model M5 T=50

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	2.302	2.366	1.955	2.267	2.213	2.058	2.723	2.340
Var	3	5.089	6.345	4.175	5.787	7.152	6.770	7.445	6.641
	8	11.795	15.201	11.495	14.979	21.805	20.878	19.311	18.193
	1	4.534	4.591	4.001	4.003	4.545	4.673	3.939	3.958
Len1	3	8.553	8.311	6.851	6.593	8.701	8.980	6.527	6.624
	8	14.860	13.284	11.892	10.548	14.864	15.624	9.938	10.183
	1	4.598	4.654	4.038	4.064	4.603	4.734	3.994	4.027
Len2	3	8.625	8.404	6.907	6.620	8.790	9.067	6.614	6.693
	8	15.074	13.512	11.921	10.561	15.026	15.812	10.043	10.307
Chi	1	2.409	3.293	2.637	4.133	3.442	3.692	3.090	3.150
Var	3	5.474	7.104	4.931	7.116	7.131	6.999	8.634	8.518
	8	11.597	16.140	10.943	14.863	20.326	18.961	20.495	20.439
	1	4.590	4.635	4.022	4.028	4.695	4.813	4.030	4.048
Len1	3	8.540	8.301	6.865	6.628	8.869	9.131	6.655	6.751
	8	14.875	13.199	11.973	10.666	15.029	15.766	10.150	10.366
	1	4.545	4.590	3.983	4.016	4.629	4.745	3.988	4.030
Len2	3	8.522	8.229	6.836	6.583	8.810	9.014	6.627	6.708
	8	14.919	13.241	12.016	10.682	15.076	15.778	10.221	10.426
Exp	1	3.315	4.513	3.458	5.333	3.945	4.055	3.013	3.074
Var	3	6.649	8.826	7.062	9.247	8.312	8.959	9.097	8.458
	8	12.654	16.103	12.695	16.307	22.534	22.214	21.510	20.873
	1	4.613	4.655	4.032	4.049	4.778	4.891	4.103	4.147
Len1	3	8.514	8.245	6.930	6.675	8.886	9.164	6.827	6.928
	8	15.057	13.283	12.184	10.801	15.032	15.858	10.464	10.711
	1	4.556	4.601	4.042	4.066	4.699	4.813	4.085	4.116
Len2	3	8.268	8.029	6.853	6.558	8.602	8.833	6.697	6.763
	8	14.245	12.696	11.882	10.452	14.175	14.901	10.103	10.264
t-dist	1	1.947	2.065	2.012	2.266	1.936	1.784	2.591	2.315
Var	3	4.954	6.169	4.957	6.530	6.840	6.560	9.204	7.855
	8	12.674	16.233	12.680	16.973	21.273	20.082	22.656	21.287
	1	4.665	4.739	4.181	4.188	4.766	4.905	4.150	4.186
Len1	3	8.515	8.248	6.976	6.674	8.827	9.123	6.723	6.783
	8	14.683	12.979	12.036	10.654	14.755	15.568	10.074	10.269
	1	4.625	4.641	4.079	4.094	4.696	4.777	4.050	4.103
Len2	3	8.486	8.224	6.921	6.640	8.744	9.001	6.641	6.730
	8	14.777	13.078	12.071	10.692	14.716	15.426	10.044	10.299

Table B.21. Variance and Lengths of Intervals for Model M1 T=100

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	1.100	1.122	1.148	1.062	1.037	0.984	1.066	1.028
Var	3	0.967	1.053	1.053	0.996	1.074	1.173	1.240	1.165
	8	1.140	1.064	1.079	1.090	1.227	1.112	1.229	1.245
	1	3.980	3.978	3.975	3.981	3.965	3.965	3.967	3.969
Lenl	3	4.610	4.639	4.612	4.626	4.540	4.536	4.542	4.543
	8	4.682	4.712	4.690	4.698	4.574	4.571	4.578	4.580
	1	4.012	4.003	3.994	4.005	3.998	3.989	3.982	3.993
Len2	3	5.316	5.329	5.312	5.270	5.171	5.176	5.176	5.163
	8	5.469	5.500	5.460	5.416	5.262	5.261	5.260	5.261
Chi	1	1.969	1.773	1.919	1.913	1.544	1.524	1.689	1.649
Var	3	1.090	1.072	1.099	1.121	1.464	1.436	1.677	1.584
	8	1.193	1.156	1.169	1.166	1.624	1.578	1.780	1.723
	1	3.904	3.926	3.918	3.907	3.970	3.984	3.915	3.913
Lenl	3	4.733	4.774	4.747	4.773	4.717	4.741	4.670	4.691
	8	4.816	4.847	4.829	4.844	4.763	4.762	4.717	4.716
	1	3.934	3.939	3.947	3.951	3.975	3.981	3.945	3.950
Len2	3	5.301	5.326	5.296	5.259	5.215	5.229	5.178	5.183
	8	5.467	5.500	5.456	5.410	5.315	5.322	5.263	5.262
Exp	1	2.430	2.343	2.185	1.666	1.783	1.775	1.969	1.918
Var	3	1.338	1.267	1.239	1.299	1.855	1.767	1.994	2.038
	8	1.480	1.377	1.399	1.410	1.959	1.904	2.140	2.080
	1	3.900	3.902	3.920	3.924	4.041	4.036	3.924	3.940
Lenl	3	4.840	4.886	4.850	4.885	4.873	4.885	4.797	4.804
	8	4.911	4.943	4.914	4.911	4.893	4.903	4.808	4.794
	1	3.935	3.927	3.946	3.968	4.030	4.034	3.967	3.989
Len2	3	5.253	5.289	5.270	5.264	5.231	5.241	5.175	5.197
	8	5.444	5.448	5.451	5.391	5.353	5.323	5.292	5.279
t-dist	1	1.280	1.285	1.237	1.244	1.150	1.177	1.189	1.187
Var	3	1.181	1.078	1.084	1.043	1.294	1.350	1.405	1.309
	8	1.259	1.197	1.297	1.213	1.437	1.464	1.709	1.354
	1	4.126	4.161	4.129	4.111	4.181	4.193	4.108	4.108
Lenl	3	4.746	4.752	4.726	4.772	4.759	4.706	4.657	4.686
	8	4.823	4.817	4.804	4.825	4.784	4.760	4.710	4.697
	1	4.067	4.110	4.070	4.078	4.075	4.106	4.064	4.071
Len2	3	5.405	5.456	5.500	5.425	5.309	5.340	5.341	5.295
	8	5.624	5.645	5.621	5.601	5.423	5.448	5.387	5.426

Table B.22. Variance and Lengths of Intervals for Model M2 T=100

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	1.020	1.036	1.101	1.055	1.094	1.068	1.234	1.232
Var	3	1.148	1.194	1.149	1.237	1.469	1.422	1.562	1.641
	8	1.495	1.532	1.486	1.567	1.695	1.714	1.805	1.830
	1	3.993	3.993	3.986	3.997	3.975	3.982	3.969	3.981
Len1	3	4.703	4.691	4.691	4.622	4.563	4.549	4.543	4.546
	8	4.843	4.854	4.832	4.705	4.593	4.608	4.588	4.579
	1	3.997	4.001	4.008	4.001	3.979	3.990	3.987	3.984
Len2	3	5.871	5.847	5.844	5.763	5.679	5.672	5.662	5.666
	8	6.588	6.567	6.558	6.302	6.104	6.099	6.081	6.060
Chi	1	2.012	2.147	1.970	1.903	1.462	1.358	1.649	1.689
Var	3	1.538	1.494	1.400	1.605	2.147	1.948	2.242	2.190
	8	1.788	1.957	1.799	1.940	2.566	2.464	2.785	2.619
	1	3.923	3.922	3.933	3.930	3.989	3.990	3.936	3.935
Len1	3	4.679	4.673	4.687	4.621	4.612	4.618	4.545	4.543
	8	4.814	4.801	4.824	4.727	4.663	4.664	4.582	4.598
	1	3.963	3.946	3.960	3.941	4.001	3.996	3.959	3.940
Len2	3	5.830	5.825	5.837	5.748	5.708	5.711	5.666	5.661
	8	6.495	6.498	6.500	6.277	6.104	6.116	6.045	6.039
Exp	1	1.834	2.119	1.859	1.711	1.576	1.490	1.750	1.754
Var	3	1.802	1.882	1.991	2.015	2.368	2.156	2.613	2.488
	8	1.690	1.766	1.681	1.783	2.791	2.667	3.125	2.935
	1	3.893	3.882	3.911	3.880	4.001	4.004	3.916	3.884
Len1	3	4.601	4.612	4.629	4.575	4.604	4.617	4.491	4.500
	8	4.754	4.760	4.785	4.665	4.658	4.675	4.543	4.534
	1	3.930	3.925	3.915	3.906	4.034	4.030	3.928	3.926
Len2	3	5.798	5.797	5.807	5.741	5.713	5.731	5.650	5.661
	8	6.483	6.468	6.475	6.284	6.121	6.130	6.037	6.063
t-dist	1	1.006	0.918	0.954	1.051	0.947	0.962	0.949	1.066
Var	3	1.390	1.295	1.543	1.469	1.385	1.475	1.620	1.382
	8	1.552	1.485	1.592	1.581	1.798	1.755	1.709	1.738
	1	4.246	4.122	4.166	4.189	4.237	4.159	4.166	4.166
Len1	3	4.773	4.775	4.759	4.703	4.676	4.691	4.635	4.620
	8	4.891	4.857	4.867	4.756	4.724	4.696	4.641	4.638
	1	4.114	4.100	4.068	4.149	4.118	4.114	4.072	4.132
Len2	3	5.900	5.893	5.891	5.764	5.767	5.734	5.737	5.676
	8	6.491	6.467	6.464	6.223	6.086	6.086	6.043	6.013

Table B.23. Variance and Lengths of Intervals for Model M3 T=100

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	1.019	1.129	1.010	1.132	0.996	0.982	1.199	1.107
Var	3	1.844	2.514	1.402	2.015	1.960	2.030	2.728	2.077
	8	4.469	7.162	3.523	7.151	7.063	6.511	9.397	6.992
	1	4.121	4.175	3.998	3.997	4.114	4.194	3.968	3.987
Len1	3	7.369	7.401	6.835	6.645	7.234	7.473	6.579	6.605
	8	12.509	12.090	10.949	9.849	11.842	12.458	9.686	9.766
	1	4.033	4.059	4.006	4.005	4.018	4.051	3.996	3.987
Len2	3	6.560	6.572	6.274	6.205	6.461	6.568	6.126	6.151
	8	11.871	11.529	10.576	9.740	11.327	11.779	9.608	9.644
Chi	1	1.597	1.619	1.694	1.762	1.752	1.879	1.735	1.732
Var	3	1.998	2.346	1.721	2.315	3.042	3.059	3.188	3.021
	8	4.337	6.056	4.051	6.668	7.049	6.925	7.880	7.094
	1	4.051	4.108	3.920	3.907	4.122	4.211	3.901	3.910
Len1	3	7.305	7.299	6.769	6.591	7.275	7.453	6.520	6.578
	8	12.320	11.934	10.867	9.779	11.731	12.308	9.640	9.726
	1	4.011	4.019	3.988	3.999	4.052	4.074	3.993	3.998
Len2	3	6.495	6.518	6.243	6.173	6.457	6.570	6.125	6.143
	8	11.718	11.422	10.508	9.711	11.206	11.651	9.547	9.659
Exp	1	1.719	2.217	2.133	3.056	2.182	2.344	1.991	2.129
Var	3	2.876	3.785	2.685	4.138	3.219	3.387	3.503	3.734
	8	5.268	7.663	4.236	8.238	6.671	6.329	7.832	7.712
	1	4.005	4.061	3.841	3.864	4.142	4.221	3.846	3.867
Len1	3	7.275	7.299	6.754	6.568	7.369	7.572	6.531	6.575
	8	12.338	11.910	10.904	9.807	11.939	12.501	9.672	9.746
	1	3.973	3.980	3.953	3.951	4.068	4.073	3.982	3.967
Len2	3	6.476	6.488	6.225	6.167	6.509	6.598	6.137	6.158
	8	11.694	11.401	10.556	9.731	11.356	11.775	9.632	9.649
t-dist	1	1.139	1.187	1.393	1.310	1.066	1.096	1.162	1.250
Var	3	1.688	2.107	1.869	2.249	2.068	1.849	2.546	2.211
	8	5.117	6.735	3.490	6.520	7.141	5.816	8.184	6.343
	1	4.250	4.260	4.084	4.101	4.310	4.361	4.063	4.113
Len1	3	7.581	7.598	7.054	6.928	7.519	7.804	6.810	6.899
	8	12.633	12.148	11.173	10.145	12.116	12.703	9.896	9.984
	1	4.197	4.225	4.165	4.159	4.215	4.258	4.156	4.149
Len2	3	6.726	6.695	6.412	6.335	6.685	6.766	6.299	6.308
	8	12.042	11.725	10.851	10.033	11.603	12.094	9.831	9.927

Table B.24. Variance and Lengths of Intervals for Model M4 T=100

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	1.129	1.308	1.184	1.248	1.189	1.239	1.393	1.286
Var	3	1.393	1.760	1.155	1.897	1.828	1.700	2.069	1.605
	8	3.310	5.290	2.586	5.845	5.807	5.298	7.072	4.862
	1	4.175	4.219	3.947	3.963	4.174	4.239	3.910	3.942
Len1	3	7.848	7.796	6.924	6.786	7.829	8.003	6.688	6.750
	8	14.075	13.317	11.477	10.551	14.247	14.719	10.412	10.608
	1	4.081	4.076	4.009	4.009	4.066	4.068	3.998	3.987
Len2	3	6.769	6.735	6.299	6.258	6.720	6.794	6.175	6.194
	8	12.969	12.398	10.866	10.196	13.009	13.362	10.034	10.183
Chi	1	1.359	1.916	1.553	2.381	2.139	2.428	1.391	1.598
Var	3	1.876	2.892	1.590	3.089	2.864	2.989	2.866	2.589
	8	4.318	7.063	3.315	8.562	7.109	6.462	8.382	6.742
	1	4.191	4.213	3.934	3.941	4.281	4.316	3.916	3.932
Len1	3	7.813	7.768	6.857	6.731	7.931	8.114	6.671	6.728
	8	13.980	13.251	11.412	10.596	14.340	14.816	10.389	10.649
	1	4.066	4.077	4.001	4.029	4.115	4.131	4.006	4.022
Len2	3	6.742	6.729	6.276	6.240	6.775	6.857	6.190	6.222
	8	12.866	12.354	10.790	10.234	13.045	13.387	10.047	10.250
Exp	1	2.238	3.242	2.177	4.007	2.276	2.767	1.626	1.806
Var	3	2.568	3.713	2.258	4.893	3.543	3.863	2.871	3.085
	8	5.084	8.498	3.507	8.965	7.017	6.622	8.148	6.870
	1	4.066	4.101	3.803	3.814	4.194	4.249	3.798	3.832
Len1	3	7.661	7.612	6.742	6.645	7.839	8.014	6.567	6.673
	8	13.736	12.968	11.222	10.378	14.029	14.494	10.239	10.479
	1	4.044	4.026	3.969	3.962	4.133	4.126	4.002	3.977
Len2	3	6.621	6.630	6.216	6.158	6.690	6.792	6.143	6.171
	8	12.640	12.087	10.657	10.054	12.808	13.124	9.929	10.106
t-dist	1	0.816	0.789	0.890	0.892	0.758	0.735	0.880	0.884
Var	3	1.482	1.762	1.551	2.350	2.039	1.936	2.476	1.975
	8	5.219	7.125	4.965	8.302	7.926	7.394	9.233	7.581
	1	4.276	4.311	4.091	4.057	4.318	4.354	4.058	4.031
Len1	3	7.738	7.665	7.019	6.805	7.826	7.945	6.804	6.784
	8	13.619	12.928	11.327	10.487	13.876	14.443	10.333	10.547
	1	4.258	4.226	4.175	4.195	4.307	4.271	4.182	4.188
Len2	3	6.773	6.768	6.423	6.299	6.795	6.881	6.309	6.248
	8	12.595	12.050	10.790	10.149	12.684	13.052	10.005	10.155

Table B.25. Variance and Lengths of Intervals for Model M5 T=100

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	1.078	1.191	0.958	1.114	0.901	0.878	1.167	1.020
Var	3	1.978	2.637	1.576	2.298	1.959	1.736	2.453	2.035
	8	5.200	7.371	4.410	6.835	7.210	6.571	8.965	7.379
	1	4.267	4.300	3.963	3.978	4.265	4.331	3.936	3.962
Len1	3	8.025	7.908	6.853	6.687	8.073	8.207	6.693	6.737
	8	14.299	13.373	11.390	10.514	14.392	14.740	10.468	10.569
	1	4.258	4.296	3.956	3.963	4.255	4.327	3.930	3.943
Len2	3	7.933	7.836	6.810	6.635	7.979	8.137	6.657	6.689
	8	13.931	13.136	11.192	10.354	14.019	14.428	10.272	10.381
Chi	1	1.518	1.956	1.694	2.372	2.386	2.637	1.620	1.640
Var	3	2.874	3.599	2.554	4.070	3.502	3.751	3.603	3.254
	8	6.232	8.239	6.131	9.170	9.624	9.429	10.143	9.095
	1	4.304	4.313	3.917	3.956	4.377	4.413	3.941	3.981
Len1	3	8.045	7.947	6.816	6.640	8.232	8.381	6.741	6.773
	8	14.310	13.449	11.352	10.490	14.712	15.143	10.596	10.693
	1	4.304	4.325	3.945	3.951	4.359	4.412	3.965	3.981
Len2	3	8.049	7.939	6.820	6.663	8.204	8.336	6.741	6.790
	8	14.356	13.433	11.342	10.499	14.685	15.002	10.588	10.669
Exp	1	2.829	3.476	2.731	3.683	2.380	2.827	1.489	1.540
Var	3	5.041	6.100	4.645	5.831	4.482	5.263	4.108	4.122
	8	8.761	10.913	7.743	9.453	11.678	11.376	11.493	11.022
	1	4.375	4.402	4.026	4.010	4.512	4.570	4.089	4.094
Len1	3	8.160	8.058	6.967	6.801	8.450	8.617	6.962	7.004
	8	14.332	13.412	11.564	10.696	14.757	15.098	10.847	10.936
	1	4.293	4.317	3.942	3.938	4.395	4.451	3.987	4.007
Len2	3	8.027	7.923	6.845	6.681	8.262	8.420	6.802	6.841
	8	14.226	13.297	11.429	10.582	14.585	14.976	10.669	10.797
t-dist	1	1.409	1.524	1.173	1.526	1.249	1.208	1.247	1.316
Var	3	3.148	3.738	2.719	4.094	3.101	2.988	3.261	3.001
	8	6.814	9.524	6.240	10.264	11.542	11.330	10.384	9.977
	1	4.350	4.390	4.062	4.029	4.416	4.469	4.064	4.046
Len1	3	8.191	8.038	6.904	6.784	8.377	8.445	6.843	6.905
	8	14.494	13.529	11.472	10.584	14.967	15.162	10.651	10.690
	1	4.411	4.420	4.045	4.055	4.445	4.489	4.064	4.084
Len2	3	8.244	8.075	6.906	6.782	8.430	8.520	6.803	6.937
	8	14.552	13.677	11.380	10.593	15.171	15.416	10.711	10.836

Table B.26. Variance and Lengths of Intervals for Model M1 T=200

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	0.549	0.586	0.610	0.566	0.566	0.634	0.623	0.614
Var	3	0.625	0.602	0.624	0.590	0.638	0.645	0.649	0.629
	8	0.660	0.624	0.604	0.598	0.678	0.663	0.652	0.633
	1	3.962	3.952	3.955	3.958	3.958	3.943	3.945	3.946
Len1	3	4.573	4.590	4.555	4.580	4.538	4.540	4.522	4.534
	8	4.622	4.629	4.609	4.623	4.575	4.568	4.563	4.567
	1	3.953	3.955	3.958	3.963	3.947	3.946	3.951	3.954
Len2	3	5.227	5.228	5.210	5.190	5.160	5.152	5.148	5.140
	8	5.319	5.331	5.318	5.303	5.218	5.220	5.220	5.221
Chi	1	1.105	1.121	1.161	1.078	0.850	0.831	0.863	0.884
Var	3	0.730	0.706	0.724	0.759	0.909	0.888	0.915	0.893
	8	0.806	0.816	0.795	0.819	0.866	0.868	0.899	0.862
	1	3.814	3.832	3.835	3.828	3.854	3.868	3.835	3.830
Len1	3	4.620	4.642	4.633	4.631	4.616	4.622	4.601	4.590
	8	4.679	4.705	4.664	4.689	4.659	4.669	4.619	4.633
	1	3.865	3.871	3.869	3.865	3.893	3.898	3.871	3.869
Len2	3	5.139	5.141	5.135	5.128	5.101	5.095	5.083	5.083
	8	5.255	5.263	5.261	5.240	5.192	5.186	5.184	5.173
Exp	1	1.226	1.347	1.227	1.163	0.841	0.826	0.901	0.887
Var	3	0.847	0.847	0.839	0.831	0.988	0.960	1.049	1.084
	8	0.872	0.847	0.861	0.848	1.100	1.062	1.135	1.101
	1	3.780	3.777	3.788	3.787	3.860	3.849	3.799	3.793
Len1	3	4.732	4.733	4.743	4.725	4.746	4.746	4.703	4.689
	8	4.754	4.778	4.745	4.768	4.752	4.759	4.696	4.709
	1	3.850	3.838	3.840	3.855	3.912	3.895	3.852	3.869
Len2	3	5.138	5.149	5.147	5.120	5.124	5.134	5.092	5.083
	8	5.264	5.279	5.260	5.246	5.231	5.225	5.186	5.181
t-dist	1	0.630	0.586	0.629	0.619	0.603	0.588	0.607	0.599
Var	3	0.669	0.652	0.636	0.679	0.708	0.687	0.654	0.723
	8	0.621	0.638	0.630	0.638	0.644	0.674	0.675	0.700
	1	4.061	4.064	4.055	4.064	4.083	4.095	4.051	4.058
Len1	3	4.651	4.665	4.643	4.661	4.660	4.656	4.610	4.628
	8	4.705	4.723	4.726	4.729	4.696	4.704	4.685	4.670
	1	4.043	4.031	4.032	4.021	4.062	4.054	4.031	4.026
Len2	3	5.298	5.315	5.297	5.284	5.257	5.262	5.229	5.243
	8	5.409	5.412	5.408	5.379	5.337	5.325	5.315	5.314

Table B.27. Variance and Lengths of Intervals for Model M2 T=200

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	0.618	0.579	0.611	0.593	0.575	0.538	0.606	0.611
Var	3	0.591	0.601	0.607	0.608	0.626	0.636	0.634	0.649
	8	0.824	0.843	0.804	0.804	0.907	0.927	0.856	0.855
	1	3.954	3.977	3.954	3.959	3.945	3.968	3.944	3.951
Lenl	3	4.587	4.582	4.598	4.558	4.513	4.509	4.519	4.515
	8	4.654	4.655	4.652	4.629	4.546	4.545	4.540	4.567
	1	3.956	3.967	3.953	3.966	3.952	3.952	3.945	3.960
Len2	3	5.710	5.723	5.730	5.682	5.620	5.632	5.634	5.630
	8	6.256	6.252	6.250	6.151	6.033	6.035	6.025	6.030
Chi	1	1.073	1.086	1.136	1.074	0.779	0.746	0.782	0.774
Var	3	0.898	0.861	0.918	0.948	1.124	1.092	1.177	1.133
	8	1.122	1.123	1.086	1.084	1.348	1.365	1.412	1.330
	1	3.805	3.805	3.811	3.796	3.840	3.838	3.812	3.804
Lenl	3	4.517	4.518	4.510	4.496	4.489	4.493	4.450	4.462
	8	4.615	4.607	4.602	4.557	4.557	4.540	4.491	4.492
	1	3.888	3.907	3.887	3.900	3.917	3.934	3.887	3.901
Len2	3	5.704	5.681	5.673	5.624	5.642	5.624	5.591	5.585
	8	6.220	6.223	6.222	6.121	6.023	6.029	6.003	6.003
Exp	1	1.538	1.541	1.623	1.451	1.010	0.987	1.022	1.062
Var	3	1.244	1.223	1.328	1.384	1.316	1.263	1.381	1.380
	8	1.178	1.179	1.132	1.236	1.532	1.483	1.613	1.573
	1	3.794	3.784	3.801	3.789	3.869	3.864	3.809	3.802
Lenl	3	4.513	4.490	4.536	4.489	4.533	4.514	4.475	4.456
	8	4.604	4.593	4.597	4.566	4.588	4.577	4.500	4.512
	1	3.844	3.846	3.839	3.852	3.903	3.904	3.846	3.860
Len2	3	5.641	5.646	5.648	5.611	5.613	5.612	5.572	5.574
	8	6.176	6.185	6.204	6.110	6.021	6.024	5.996	5.987
t-dist	1	0.545	0.529	0.543	0.557	0.501	0.481	0.490	0.497
Var	3	0.716	0.757	0.777	0.796	0.767	0.763	0.793	0.793
	8	0.921	0.945	0.925	0.922	0.944	1.004	0.983	0.971
	1	4.066	4.076	4.079	4.078	4.101	4.104	4.080	4.076
Lenl	3	4.663	4.678	4.679	4.644	4.634	4.644	4.615	4.614
	8	4.753	4.752	4.752	4.731	4.679	4.678	4.641	4.666
	1	4.032	4.032	4.029	4.036	4.048	4.062	4.028	4.038
Len2	3	5.782	5.780	5.800	5.748	5.721	5.729	5.721	5.709
	8	6.286	6.292	6.308	6.199	6.098	6.106	6.093	6.095

Table B.28. Variance and Lengths of Intervals for Model M3 T=200

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	0.606	0.614	0.619	0.608	0.611	0.591	0.637	0.615
Var	3	0.720	0.909	0.676	0.819	0.781	0.778	0.870	0.752
	8	1.810	2.426	1.443	2.262	2.298	2.392	2.513	2.304
	1	4.022	4.034	3.986	3.983	4.015	4.037	3.971	3.977
Len1	3	6.952	6.976	6.773	6.647	6.832	6.923	6.623	6.635
	8	11.301	11.249	10.708	10.027	10.733	11.036	9.957	9.983
	1	3.968	3.979	3.972	3.964	3.965	3.972	3.963	3.956
Len2	3	6.249	6.262	6.172	6.106	6.186	6.232	6.099	6.089
	8	10.797	10.782	10.335	9.836	10.355	10.604	9.774	9.801
Chi	1	0.901	0.882	1.364	1.178	1.117	1.137	1.062	1.134
Var	3	0.734	0.876	0.892	1.003	1.469	1.486	1.466	1.654
	8	1.662	2.233	1.770	2.429	2.722	2.629	2.931	2.780
	1	3.909	3.931	3.846	3.863	3.956	3.988	3.844	3.867
Len1	3	6.906	6.940	6.696	6.594	6.884	6.969	6.562	6.587
	8	11.285	11.269	10.679	9.998	10.822	11.185	9.955	9.968
	1	3.929	3.933	3.925	3.942	3.956	3.960	3.934	3.944
Len2	3	6.244	6.253	6.147	6.107	6.217	6.268	6.094	6.101
	8	10.821	10.799	10.335	9.837	10.476	10.705	9.798	9.810
Exp	1	1.388	1.347	3.058	2.546	1.054	1.155	1.066	1.158
Var	3	1.128	1.270	1.846	1.916	1.805	1.860	1.776	2.091
	8	1.944	2.696	2.349	2.966	3.076	2.986	3.317	3.483
	1	3.879	3.912	3.812	3.813	3.963	4.015	3.814	3.825
Len1	3	6.825	6.864	6.620	6.510	6.853	6.966	6.491	6.510
	8	11.118	11.096	10.530	9.864	10.734	11.070	9.831	9.838
	1	3.867	3.879	3.860	3.865	3.919	3.941	3.876	3.880
Len2	3	6.174	6.205	6.110	6.049	6.187	6.244	6.062	6.065
	8	10.684	10.713	10.242	9.735	10.388	10.660	9.737	9.721
t-dist	1	0.545	0.545	0.603	0.576	0.521	0.495	0.601	0.569
Var	3	0.674	0.756	0.663	0.751	0.709	0.703	0.779	0.771
	8	1.672	2.277	1.656	2.353	2.257	2.274	2.596	2.306
	1	4.120	4.139	4.061	4.063	4.142	4.178	4.057	4.066
Len1	3	7.102	7.123	6.919	6.791	7.051	7.148	6.784	6.784
	8	11.480	11.425	10.871	10.199	11.008	11.328	10.133	10.173
	1	4.075	4.071	4.067	4.071	4.090	4.108	4.072	4.070
Len2	3	6.367	6.367	6.278	6.225	6.337	6.385	6.226	6.213
	8	10.974	10.932	10.480	9.979	10.595	10.851	9.945	9.950

Table B.29. Variance and Lengths of Intervals for Model M4 T=200

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	0.599	0.620	0.578	0.599	0.576	0.575	0.616	0.607
Var	3	0.692	0.818	0.539	0.678	0.708	0.677	0.659	0.618
	8	1.686	2.549	1.112	2.407	2.197	1.935	2.243	1.623
	1	4.047	4.054	3.934	3.940	4.039	4.065	3.914	3.932
Len1	3	7.332	7.298	6.828	6.758	7.287	7.357	6.710	6.750
	8	12.836	12.492	11.207	10.656	12.728	12.992	10.624	10.714
	1	3.996	3.997	3.977	3.987	3.990	3.991	3.967	3.974
Len2	3	6.387	6.403	6.179	6.146	6.356	6.404	6.113	6.120
	8	11.885	11.632	10.611	10.220	11.776	11.944	10.173	10.237
Chi	1	0.963	1.352	1.039	1.761	1.123	1.359	0.785	0.875
Var	3	1.097	1.588	0.867	1.812	1.656	1.925	1.015	1.250
	8	1.937	3.167	1.664	4.176	2.709	2.553	2.447	2.121
	1	3.985	4.015	3.831	3.851	4.034	4.071	3.826	3.852
Len1	3	7.298	7.321	6.757	6.712	7.339	7.446	6.658	6.709
	8	12.883	12.668	11.155	10.678	12.904	13.269	10.613	10.760
	1	3.932	3.944	3.894	3.906	3.961	3.973	3.902	3.907
Len2	3	6.363	6.382	6.137	6.134	6.369	6.426	6.099	6.123
	8	11.895	11.711	10.543	10.217	11.882	12.108	10.166	10.269
Exp	1	1.459	2.171	1.606	3.296	1.681	1.985	0.999	1.162
Var	3	1.832	2.718	1.572	3.314	2.420	2.803	1.374	1.737
	8	3.006	4.463	2.371	5.338	3.461	3.373	3.135	2.921
	1	3.995	4.034	3.826	3.841	4.079	4.133	3.833	3.849
Len1	3	7.328	7.311	6.727	6.632	7.445	7.535	6.652	6.649
	8	12.985	12.689	11.084	10.659	13.173	13.483	10.618	10.738
	1	3.903	3.919	3.886	3.899	3.960	3.986	3.907	3.903
Len2	3	6.379	6.372	6.110	6.074	6.415	6.459	6.098	6.082
	8	11.950	11.757	10.506	10.189	12.064	12.277	10.168	10.236
t-dist	1	0.584	0.657	0.586	0.633	0.567	0.610	0.579	0.577
Var	3	0.738	0.917	0.742	0.939	0.766	0.774	0.916	0.781
	8	1.750	2.615	1.343	2.691	2.119	1.977	2.440	1.952
	1	4.110	4.137	3.992	4.002	4.140	4.175	3.988	4.001
Len1	3	7.435	7.439	6.920	6.844	7.457	7.563	6.821	6.851
	8	13.010	12.728	11.226	10.741	13.094	13.402	10.702	10.842
	1	4.050	4.071	4.014	4.024	4.070	4.084	4.016	4.022
Len2	3	6.431	6.432	6.201	6.163	6.435	6.473	6.157	6.166
	8	11.971	11.740	10.595	10.275	12.021	12.184	10.217	10.319

Table B.30. Variance and Lengths of Intervals for Model M5 T=200

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	0.585	0.595	0.612	0.606	0.562	0.509	0.663	0.559
Var	3	1.067	1.280	0.781	1.060	1.030	0.920	1.161	0.981
	8	3.048	3.848	2.336	3.660	3.871	3.515	4.340	3.739
	1	4.145	4.159	3.962	3.973	4.145	4.170	3.948	3.963
Len1	3	7.632	7.566	6.815	6.743	7.650	7.707	6.743	6.779
	8	13.603	13.182	11.184	10.714	13.807	13.991	10.797	10.839
	1	4.117	4.133	3.944	3.931	4.112	4.147	3.930	3.927
Len2	3	7.566	7.528	6.802	6.720	7.578	7.656	6.720	6.758
	8	13.428	13.012	11.088	10.587	13.542	13.755	10.684	10.747
Chi	1	1.032	1.379	1.020	1.557	1.601	1.846	0.907	0.974
Var	3	1.383	1.846	1.274	1.864	2.243	2.476	1.853	1.739
	8	3.121	4.238	2.682	4.294	4.480	4.570	5.182	4.309
	1	4.120	4.113	3.891	3.899	4.166	4.172	3.905	3.921
Len1	3	7.580	7.549	6.770	6.721	7.668	7.747	6.751	6.811
	8	13.430	13.029	11.116	10.613	13.646	13.842	10.808	10.846
	1	4.111	4.136	3.910	3.923	4.152	4.183	3.923	3.936
Len2	3	7.599	7.536	6.797	6.714	7.689	7.724	6.764	6.785
	8	13.596	13.117	11.115	10.708	13.822	13.919	10.787	10.874
Exp	1	0.921	1.710	1.033	2.243	1.784	2.079	0.952	0.995
Var	3	1.796	2.481	1.812	3.121	2.680	2.986	1.945	1.987
	8	3.222	4.738	2.912	5.117	4.693	4.564	4.306	4.360
	1	4.101	4.087	3.852	3.874	4.183	4.176	3.887	3.915
Len1	3	7.516	7.446	6.741	6.646	7.667	7.703	6.747	6.772
	8	13.250	12.851	11.069	10.557	13.542	13.728	10.783	10.842
	1	4.095	4.107	3.887	3.889	4.163	4.190	3.917	3.931
Len2	3	7.573	7.527	6.777	6.682	7.701	7.764	6.797	6.811
	8	13.494	13.071	11.137	10.616	13.767	13.965	10.906	10.921
t-dist	1	0.539	0.593	0.633	0.628	0.498	0.499	0.677	0.614
Var	3	1.004	1.163	0.812	1.044	0.917	0.844	1.100	1.046
	8	2.451	3.219	2.218	3.105	2.720	2.646	3.609	2.919
	1	4.224	4.255	4.060	4.054	4.256	4.297	4.070	4.066
Len1	3	7.701	7.686	6.946	6.838	7.780	7.879	6.920	6.929
	8	13.605	13.239	11.265	10.767	13.869	14.133	10.964	10.993
	1	4.211	4.211	4.032	4.031	4.238	4.249	4.040	4.043
Len2	3	7.672	7.640	6.914	6.831	7.741	7.835	6.881	6.913
	8	13.436	13.108	11.192	10.729	13.698	14.001	10.869	10.932

Table B.31. Average Coverage Probability for Model M6 T=50

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.8876	0.8990	0.9032	0.9124	0.9165	0.9191	0.9243	0.9244
PRRP:	0.8848	0.8991	0.8996	0.9093	0.9116	0.9133	0.9184	0.9191
KIM:	0.8822	0.8951	0.8999	0.9065	0.9121	0.9160	0.9209	0.9201
KIMP:	0.8841	0.8995	0.9029	0.9097	0.9146	0.9167	0.9197	0.9194
PRR%:	0.8732	0.8803	0.8749	0.8745	0.8713	0.8711	0.8774	0.8757
PRRP%:	0.8778	0.8832	0.8771	0.8767	0.8750	0.8752	0.8790	0.8761
KIM%:	0.8551	0.8669	0.8610	0.8631	0.8614	0.8597	0.8610	0.8610
KIMP%:	0.8735	0.8782	0.8729	0.8744	0.8708	0.8707	0.8753	0.8749

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9035	0.9066	0.9130	0.9172	0.9221	0.9260	0.9304	0.9303
PRRP:	0.8996	0.9045	0.9099	0.9152	0.9181	0.9209	0.9238	0.9235
KIM:	0.8945	0.8930	0.9007	0.9031	0.9113	0.9158	0.9199	0.9246
KIMP:	0.8990	0.8995	0.9088	0.9135	0.9186	0.9212	0.9229	0.9260
PRR%:	0.8516	0.8725	0.8828	0.8801	0.8801	0.8808	0.8843	0.8839
PRRP%:	0.8515	0.8705	0.8792	0.8768	0.8785	0.8804	0.8823	0.8839
KIM%:	0.8305	0.8551	0.8619	0.8593	0.8606	0.8616	0.8632	0.8656
KIMP%:	0.8388	0.8600	0.8658	0.8645	0.8671	0.8670	0.8697	0.8708

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9202	0.9009	0.9058	0.9081	0.9095	0.9112	0.9129	0.9153
PRRP:	0.9239	0.9048	0.9063	0.9059	0.9084	0.9084	0.9088	0.9102
KIM:	0.9156	0.9025	0.9027	0.9053	0.9079	0.9095	0.9123	0.9116
KIMP:	0.9238	0.9049	0.9069	0.9083	0.9102	0.9097	0.9117	0.9116
PRR%:	0.8370	0.8634	0.8724	0.8741	0.8749	0.8731	0.8767	0.8761
PRRP%:	0.8366	0.8628	0.8704	0.8703	0.8723	0.8714	0.8730	0.8726
KIM%:	0.8167	0.8448	0.8514	0.8470	0.8505	0.8475	0.8494	0.8491
KIMP%:	0.8199	0.8430	0.8545	0.8536	0.8558	0.8551	0.8569	0.8540

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8783	0.8932	0.8991	0.9052	0.9115	0.9141	0.9172	0.9184
PRRP:	0.8815	0.8906	0.8939	0.9021	0.9067	0.9093	0.9095	0.9109
KIM:	0.8739	0.8878	0.8924	0.9026	0.9082	0.9128	0.9152	0.9162
KIMP:	0.8813	0.8911	0.8977	0.9038	0.9092	0.9111	0.9113	0.9131
PRR%:	0.8837	0.8852	0.8836	0.8830	0.8827	0.8848	0.8845	0.8862
PRRP%:	0.8823	0.8842	0.8822	0.8840	0.8842	0.8839	0.8843	0.8874
KIM%:	0.8704	0.8725	0.8687	0.8680	0.8696	0.8702	0.8710	0.8721
KIMP%:	0.8805	0.8764	0.8739	0.8759	0.8785	0.8791	0.8792	0.8796

Table B.32. Average Coverage Probability for Model M6 T=100

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.8914	0.9014	0.9053	0.9098	0.9090	0.9082	0.9131	0.9125
PRRP:	0.8918	0.9025	0.9040	0.9092	0.9101	0.9082	0.9105	0.9100
KIM:	0.8900	0.9020	0.9048	0.9086	0.9097	0.9090	0.9128	0.9113
KIMP:	0.8919	0.9028	0.9040	0.9085	0.9091	0.9110	0.9134	0.9115
PRR%:	0.8862	0.8918	0.8901	0.8900	0.8858	0.8840	0.8876	0.8866
PRRP%:	0.8881	0.8912	0.8875	0.8902	0.8873	0.8874	0.8872	0.8858
KIM%:	0.8778	0.8856	0.8838	0.8843	0.8805	0.8788	0.8827	0.8814
KIMP%:	0.8861	0.8900	0.8879	0.8885	0.8864	0.8869	0.8902	0.8879

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9046	0.9001	0.8975	0.9036	0.9050	0.9087	0.9101	0.9108
PRRP:	0.9097	0.8997	0.8972	0.9057	0.9027	0.9042	0.9057	0.9074
KIM:	0.9033	0.8961	0.8962	0.9031	0.9057	0.9073	0.9087	0.9075
KIMP:	0.9066	0.9003	0.9007	0.9051	0.9068	0.9073	0.9090	0.9106
PRR%:	0.8470	0.8732	0.8788	0.8799	0.8797	0.8808	0.8813	0.8809
PRRP%:	0.8483	0.8721	0.8771	0.8811	0.8796	0.8787	0.8793	0.8793
KIM%:	0.8360	0.8618	0.8638	0.8667	0.8672	0.8670	0.8669	0.8648
KIMP%:	0.8414	0.8646	0.8712	0.8728	0.8730	0.8721	0.8729	0.8752

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9202	0.9009	0.9058	0.9081	0.9095	0.9112	0.9129	0.9153
PRRP:	0.9239	0.9048	0.9063	0.9059	0.9084	0.9084	0.9088	0.9102
KIM:	0.9156	0.9025	0.9027	0.9053	0.9079	0.9095	0.9123	0.9116
KIMP:	0.9238	0.9049	0.9069	0.9083	0.9102	0.9097	0.9117	0.9116
PRR%:	0.8370	0.8634	0.8724	0.8741	0.8749	0.8731	0.8767	0.8761
PRRP%:	0.8366	0.8628	0.8704	0.8703	0.8723	0.8714	0.8730	0.8726
KIM%:	0.8167	0.8448	0.8514	0.8470	0.8505	0.8475	0.8494	0.8491
KIMP%:	0.8199	0.8430	0.8545	0.8536	0.8558	0.8551	0.8569	0.8540

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8923	0.9003	0.9000	0.9039	0.9073	0.9079	0.9093	0.9099
PRRP:	0.8924	0.9020	0.9022	0.9045	0.9062	0.9061	0.9060	0.9069
KIM:	0.8894	0.9015	0.8996	0.9029	0.9055	0.9080	0.9097	0.9092
KIMP:	0.8938	0.9020	0.9017	0.9054	0.9066	0.9069	0.9072	0.9068
PRR%:	0.8965	0.8963	0.8936	0.8931	0.8952	0.8941	0.8921	0.8937
PRRP%:	0.8957	0.8980	0.8941	0.8944	0.8948	0.8917	0.8913	0.8929
KIM%:	0.8873	0.8913	0.8864	0.8849	0.8870	0.8860	0.8874	0.8856
KIMP%:	0.8930	0.8949	0.8890	0.8892	0.8891	0.8894	0.8902	0.8892

Table B.33. Average Coverage Probability for Model M6 T=200

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.8914	0.9014	0.9053	0.9098	0.9090	0.9082	0.9131	0.9125
PRRP:	0.8918	0.9025	0.9040	0.9092	0.9101	0.9082	0.9105	0.9100
KIM:	0.8900	0.9020	0.9048	0.9086	0.9097	0.9090	0.9128	0.9113
KIMP:	0.8919	0.9028	0.9040	0.9085	0.9091	0.9110	0.9134	0.9115
PRR%:	0.8862	0.8918	0.8901	0.8900	0.8858	0.8840	0.8876	0.8866
PRRP%:	0.8881	0.8912	0.8875	0.8902	0.8873	0.8874	0.8872	0.8858
KIM%:	0.8778	0.8856	0.8838	0.8843	0.8805	0.8788	0.8827	0.8814
KIMP%:	0.8861	0.8900	0.8879	0.8885	0.8864	0.8869	0.8902	0.8879

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9046	0.9001	0.8975	0.9036	0.9050	0.9087	0.9101	0.9108
PRRP:	0.9095	0.8993	0.8969	0.9055	0.9025	0.9042	0.9054	0.9074
KIM:	0.9038	0.8965	0.8961	0.9028	0.9045	0.9059	0.9075	0.9061
KIMP:	0.9066	0.9003	0.9007	0.9051	0.9068	0.9073	0.9090	0.9106
PRR%:	0.8470	0.8732	0.8788	0.8799	0.8797	0.8808	0.8813	0.8809
PRRP%:	0.8480	0.8720	0.8772	0.8809	0.8796	0.8788	0.8794	0.8794
KIM%:	0.8362	0.8615	0.8639	0.8672	0.8676	0.8675	0.8671	0.8649
KIMP%:	0.8414	0.8646	0.8712	0.8728	0.8730	0.8721	0.8729	0.8752

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9265	0.9153	0.9155	0.9146	0.9165	0.9163	0.9175	0.9200
PRRP:	0.9274	0.9160	0.9170	0.9165	0.9164	0.9163	0.9166	0.9186
KIM:	0.9193	0.9125	0.9132	0.9141	0.9149	0.9160	0.9163	0.9172
KIMP:	0.9295	0.9158	0.9166	0.9148	0.9177	0.9157	0.9187	0.9180
PRR%:	0.8213	0.8663	0.8792	0.8821	0.8834	0.8835	0.8844	0.8843
PRRP%:	0.8206	0.8648	0.8774	0.8832	0.8833	0.8833	0.8833	0.8846
KIM%:	0.8041	0.8501	0.8619	0.8654	0.8663	0.8678	0.8675	0.8681
KIMP%:	0.8088	0.8525	0.8656	0.8686	0.8720	0.8704	0.8717	0.8715

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8923	0.9003	0.9000	0.9039	0.9073	0.9079	0.9093	0.9099
PRRP:	0.8924	0.9020	0.9022	0.9045	0.9062	0.9061	0.9060	0.9069
KIM:	0.8894	0.9015	0.8996	0.9029	0.9055	0.9080	0.9097	0.9092
KIMP:	0.8938	0.9020	0.9017	0.9054	0.9066	0.9069	0.9072	0.9068
PRR%:	0.8965	0.8963	0.8936	0.8931	0.8952	0.8941	0.8921	0.8937
PRRP%:	0.8957	0.8980	0.8941	0.8944	0.8948	0.8917	0.8913	0.8929
KIM%:	0.8873	0.8913	0.8864	0.8849	0.8870	0.8860	0.8874	0.8856
KIMP%:	0.8930	0.8949	0.8890	0.8892	0.8891	0.8894	0.8902	0.8892

Table B.34. Average Coverage Probability for Model M7 T=50

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.8851	0.9017	0.9082	0.9151	0.9179	0.9219	0.9245	0.9249
PRRP:	0.8857	0.9029	0.9062	0.9131	0.9150	0.9143	0.9167	0.9196
KIM:	0.8818	0.9051	0.9089	0.9148	0.9182	0.9212	0.9240	0.9243
KIMP:	0.8866	0.9065	0.9117	0.9185	0.9202	0.9220	0.9238	0.9243
PRR%:	0.8740	0.8797	0.8793	0.8783	0.8760	0.8767	0.8782	0.8783
PRRP%:	0.8772	0.8829	0.8798	0.8787	0.8775	0.8787	0.8793	0.8798
KIM%:	0.8576	0.8654	0.8589	0.8597	0.8580	0.8592	0.8591	0.8567
KIMP%:	0.8722	0.8753	0.8729	0.8727	0.8714	0.8734	0.8725	0.8742

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9208	0.9102	0.9169	0.9166	0.9218	0.9219	0.9255	0.9256
PRRP:	0.9239	0.9107	0.9136	0.9158	0.9193	0.9214	0.9236	0.9232
KIM:	0.9152	0.9042	0.9042	0.9033	0.9124	0.9102	0.9176	0.9160
KIMP:	0.9240	0.9124	0.9140	0.9133	0.9185	0.9169	0.9203	0.9207
PRR%:	0.8624	0.8813	0.8882	0.8875	0.8855	0.8835	0.8852	0.8864
PRRP%:	0.8598	0.8775	0.8865	0.8861	0.8841	0.8827	0.8863	0.8871
KIM%:	0.8448	0.8617	0.8681	0.8694	0.8697	0.8659	0.8676	0.8688
KIMP%:	0.8496	0.8664	0.8747	0.8761	0.8757	0.8731	0.8750	0.8748

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9096	0.9076	0.9064	0.9105	0.9147	0.9170	0.9187	0.9202
PRRP:	0.9101	0.9077	0.9064	0.9076	0.9115	0.9130	0.9132	0.9142
KIM:	0.8954	0.8948	0.8905	0.8978	0.9029	0.9071	0.9086	0.9116
KIMP:	0.9110	0.9048	0.9050	0.9063	0.9087	0.9102	0.9130	0.9132
PRR%:	0.8496	0.8658	0.8679	0.8703	0.8723	0.8750	0.8777	0.8800
PRRP%:	0.8494	0.8623	0.8653	0.8664	0.8699	0.8719	0.8733	0.8762
KIM%:	0.8241	0.8387	0.8408	0.8417	0.8399	0.8427	0.8447	0.8430
KIMP%:	0.8334	0.8468	0.8475	0.8482	0.8502	0.8525	0.8552	0.8547

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8915	0.8931	0.8967	0.9030	0.9062	0.9065	0.9097	0.9106
PRRP:	0.8895	0.8917	0.8957	0.9002	0.9015	0.9046	0.9048	0.9073
KIM:	0.8871	0.8924	0.8977	0.9017	0.9043	0.9075	0.9088	0.9118
KIMP:	0.8905	0.8942	0.8969	0.9037	0.9065	0.9065	0.9078	0.9096
PRR%:	0.8863	0.8815	0.8781	0.8809	0.8773	0.8775	0.8793	0.8785
PRRP%:	0.8861	0.8818	0.8783	0.8794	0.8749	0.8777	0.8791	0.8808
KIM%:	0.8738	0.8705	0.8639	0.8615	0.8624	0.8635	0.8628	0.8635
KIMP%:	0.8814	0.8763	0.8714	0.8720	0.8692	0.8707	0.8724	0.8707

Table B.35. Average Coverage Probability for Model M7 T=100

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.8938	0.9068	0.9055	0.9080	0.9127	0.9166	0.9164	0.9146
PRRP:	0.8943	0.9046	0.9051	0.9083	0.9113	0.9142	0.9169	0.9142
KIM:	0.8933	0.9045	0.9020	0.9055	0.9112	0.9155	0.9157	0.9122
KIMP:	0.8932	0.9070	0.9058	0.9088	0.9120	0.9146	0.9167	0.9155
PRR%:	0.8889	0.8964	0.8893	0.8906	0.8911	0.8926	0.8924	0.8912
PRRP%:	0.8906	0.8936	0.8898	0.8900	0.8907	0.8936	0.8956	0.8929
KIM%:	0.8868	0.8926	0.8832	0.8857	0.8864	0.8886	0.8890	0.8876
KIMP%:	0.8875	0.8963	0.8880	0.8895	0.8900	0.8904	0.8952	0.8924

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9065	0.9021	0.9002	0.9031	0.9069	0.9081	0.9079	0.9098
PRRP:	0.9059	0.9027	0.9015	0.9031	0.9063	0.9056	0.9020	0.9048
KIM:	0.9080	0.9028	0.9020	0.9066	0.9086	0.9077	0.9083	0.9116
KIMP:	0.9087	0.9052	0.9020	0.9068	0.9084	0.9082	0.9074	0.9082
PRR%:	0.8386	0.8679	0.8758	0.8766	0.8780	0.8773	0.8760	0.8769
PRRP%:	0.8390	0.8678	0.8755	0.8781	0.8787	0.8775	0.8749	0.8778
KIM%:	0.8277	0.8585	0.8634	0.8663	0.8679	0.8635	0.8631	0.8660
KIMP%:	0.8315	0.8600	0.8686	0.8716	0.8700	0.8695	0.8683	0.8694

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9096	0.9076	0.9064	0.9105	0.9147	0.9170	0.9187	0.9202
PRRP:	0.9101	0.9077	0.9064	0.9076	0.9115	0.9130	0.9132	0.9142
KIM:	0.8954	0.8948	0.8905	0.8978	0.9029	0.9071	0.9086	0.9116
KIMP:	0.9110	0.9048	0.9050	0.9063	0.9087	0.9102	0.9130	0.9132
PRR%:	0.8496	0.8658	0.8679	0.8703	0.8723	0.8750	0.8777	0.8800
PRRP%:	0.8494	0.8623	0.8653	0.8664	0.8699	0.8719	0.8733	0.8762
KIM%:	0.8241	0.8387	0.8408	0.8417	0.8399	0.8427	0.8447	0.8430
KIMP%:	0.8334	0.8468	0.8475	0.8482	0.8502	0.8525	0.8552	0.8547

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8906	0.9011	0.9028	0.9025	0.9031	0.9052	0.9074	0.9077
PRRP:	0.8920	0.9010	0.9023	0.9028	0.9032	0.9031	0.9039	0.9043
KIM:	0.8895	0.8996	0.9014	0.9010	0.9018	0.9053	0.9070	0.9052
KIMP:	0.8933	0.9014	0.9038	0.9040	0.9028	0.9070	0.9060	0.9050
PRR%:	0.8935	0.8999	0.8961	0.8950	0.8940	0.8932	0.8967	0.8955
PRRP%:	0.8949	0.9018	0.8976	0.8958	0.8972	0.8952	0.8969	0.8959
KIM%:	0.8880	0.8901	0.8890	0.8880	0.8859	0.8870	0.8888	0.8866
KIMP%:	0.8915	0.8959	0.8944	0.8934	0.8924	0.8944	0.8965	0.8930

Table B.36. Average Coverage Probability for Model M7 T=200

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.8938	0.9068	0.9055	0.9080	0.9127	0.9166	0.9164	0.9146
PRRP:	0.8943	0.9046	0.9051	0.9083	0.9113	0.9142	0.9169	0.9142
KIM:	0.8933	0.9045	0.9020	0.9055	0.9112	0.9155	0.9157	0.9122
KIMP:	0.8932	0.9070	0.9058	0.9088	0.9120	0.9146	0.9167	0.9155
PRR%:	0.8889	0.8964	0.8893	0.8906	0.8911	0.8926	0.8924	0.8912
PRRP%:	0.8906	0.8936	0.8898	0.8900	0.8907	0.8936	0.8956	0.8929
KIM%:	0.8868	0.8926	0.8832	0.8857	0.8864	0.8886	0.8890	0.8876
KIMP%:	0.8875	0.8963	0.8880	0.8895	0.8900	0.8904	0.8952	0.8924

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9065	0.9021	0.9002	0.9031	0.9069	0.9081	0.9079	0.9098
PRRP:	0.9056	0.9026	0.9015	0.9031	0.9061	0.9058	0.9021	0.9050
KIM:	0.9093	0.9032	0.9019	0.9063	0.9079	0.9070	0.9064	0.9095
KIMP:	0.9087	0.9052	0.9020	0.9068	0.9084	0.9082	0.9074	0.9082
PRR%:	0.8386	0.8679	0.8758	0.8766	0.8780	0.8773	0.8760	0.8769
PRRP%:	0.8388	0.8678	0.8755	0.8783	0.8788	0.8775	0.8748	0.8780
KIM%:	0.8285	0.8585	0.8636	0.8665	0.8682	0.8639	0.8635	0.8666
KIMP%:	0.8315	0.8600	0.8686	0.8716	0.8700	0.8695	0.8683	0.8694

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9261	0.9079	0.9084	0.9095	0.9089	0.9115	0.9127	0.9150
PRRP:	0.9240	0.9102	0.9050	0.9072	0.9082	0.9108	0.9097	0.9126
KIM:	0.9104	0.9051	0.9044	0.9069	0.9084	0.9126	0.9134	0.9153
KIMP:	0.9264	0.9110	0.9083	0.9095	0.9102	0.9130	0.9135	0.9136
PRR%:	0.8229	0.8611	0.8739	0.8771	0.8788	0.8804	0.8793	0.8824
PRRP%:	0.8217	0.8603	0.8744	0.8796	0.8791	0.8812	0.8789	0.8807
KIM%:	0.8061	0.8450	0.8618	0.8665	0.8669	0.8676	0.8658	0.8679
KIMP%:	0.8112	0.8481	0.8619	0.8682	0.8693	0.8692	0.8668	0.8698

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8906	0.9011	0.9028	0.9025	0.9031	0.9052	0.9074	0.9077
PRRP:	0.8920	0.9010	0.9023	0.9028	0.9032	0.9031	0.9039	0.9043
KIM:	0.8895	0.8996	0.9014	0.9010	0.9018	0.9053	0.9070	0.9052
KIMP:	0.8933	0.9014	0.9038	0.9040	0.9028	0.9070	0.9060	0.9050
PRR%:	0.8935	0.8999	0.8961	0.8950	0.8940	0.8932	0.8967	0.8955
PRRP%:	0.8949	0.9018	0.8976	0.8958	0.8972	0.8952	0.8969	0.8959
KIM%:	0.8880	0.8901	0.8890	0.8880	0.8859	0.8870	0.8888	0.8866
KIMP%:	0.8915	0.8959	0.8944	0.8934	0.8924	0.8944	0.8965	0.8930

Table B.37. Average Coverage Probability for Model M8 T=50

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.8827	0.8963	0.9048	0.9151	0.9160	0.9230	0.9246	0.9268
PRRP:	0.8827	0.8945	0.8983	0.9075	0.9103	0.9128	0.9128	0.9152
KIM:	0.8813	0.8941	0.9035	0.9146	0.9191	0.9234	0.9275	0.9305
KIMP:	0.8829	0.8923	0.9013	0.9071	0.9065	0.9090	0.9127	0.9152
PRR%:	0.8695	0.8748	0.8799	0.8881	0.8904	0.8927	0.8913	0.8948
PRRP%:	0.8708	0.8727	0.8755	0.8864	0.8877	0.8917	0.8902	0.8960
KIM%:	0.8656	0.8675	0.8745	0.8818	0.8838	0.8862	0.8872	0.8900
KIMP%:	0.8704	0.8745	0.8739	0.8832	0.8819	0.8875	0.8877	0.8916

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9017	0.9062	0.8975	0.9073	0.9120	0.9115	0.9187	0.9154
PRRP:	0.8998	0.9107	0.8964	0.9019	0.9044	0.9037	0.9101	0.9088
KIM:	0.9021	0.9078	0.8996	0.9085	0.9130	0.9149	0.9198	0.9197
KIMP:	0.8998	0.9116	0.8953	0.9018	0.9033	0.9052	0.9079	0.9087
PRR%:	0.8360	0.8607	0.8710	0.8784	0.8764	0.8800	0.8879	0.8832
PRRP%:	0.8358	0.8655	0.8669	0.8731	0.8699	0.8775	0.8838	0.8797
KIM%:	0.8227	0.8494	0.8677	0.8696	0.8689	0.8712	0.8780	0.8746
KIMP%:	0.8279	0.8554	0.8603	0.8673	0.8626	0.8706	0.8759	0.8741

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9131	0.9062	0.8978	0.8992	0.9062	0.9091	0.9094	0.9107
PRRP:	0.9088	0.9048	0.8946	0.8953	0.8978	0.9022	0.9020	0.9003
KIM:	0.9151	0.9025	0.8970	0.8968	0.9061	0.9106	0.9106	0.9096
KIMP:	0.9155	0.9066	0.8958	0.8926	0.8998	0.9020	0.9017	0.8992
PRR%:	0.8365	0.8460	0.8684	0.8724	0.8764	0.8741	0.8764	0.8779
PRRP%:	0.8343	0.8443	0.8667	0.8698	0.8744	0.8715	0.8729	0.8755
KIM%:	0.8180	0.8327	0.8598	0.8626	0.8667	0.8663	0.8669	0.8663
KIMP%:	0.8227	0.8312	0.8560	0.8601	0.8634	0.8616	0.8645	0.8644

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8787	0.8889	0.9040	0.9029	0.9142	0.9119	0.9171	0.9192
PRRP:	0.8807	0.8887	0.8989	0.8975	0.9030	0.9035	0.9054	0.9099
KIM:	0.8771	0.8914	0.9022	0.9050	0.9135	0.9136	0.9192	0.9193
KIMP:	0.8813	0.8885	0.8993	0.8973	0.9073	0.9031	0.9070	0.9073
PRR%:	0.8824	0.8812	0.8900	0.8890	0.8966	0.8930	0.8955	0.8967
PRRP%:	0.8835	0.8818	0.8875	0.8858	0.8929	0.8921	0.8958	0.8976
KIM%:	0.8736	0.8746	0.8795	0.8775	0.8856	0.8842	0.8865	0.8848
KIMP%:	0.8738	0.8759	0.8818	0.8792	0.8890	0.8862	0.8899	0.8893

Table B.38. Average Coverage Probability for Model M8 T=100

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.9002	0.9044	0.9123	0.9127	0.9179	0.9170	0.9166	0.9187
PRRP:	0.9014	0.9038	0.9103	0.9099	0.9127	0.9121	0.9121	0.9141
KIM:	0.8996	0.9038	0.9111	0.9145	0.9167	0.9194	0.9178	0.9191
KIMP:	0.8988	0.9025	0.9090	0.9106	0.9117	0.9119	0.9108	0.9114
PRR%:	0.8910	0.8964	0.9002	0.9008	0.9059	0.9035	0.9028	0.9046
PRRP%:	0.8924	0.8950	0.9008	0.9004	0.9057	0.9034	0.9046	0.9069
KIM%:	0.8884	0.8940	0.8977	0.9002	0.9022	0.9030	0.9019	0.9034
KIMP%:	0.8886	0.8957	0.9008	0.9009	0.9042	0.9048	0.9038	0.9034

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9166	0.9173	0.9118	0.9152	0.9155	0.9172	0.9188	0.9207
PRRP:	0.9173	0.9176	0.9089	0.9129	0.9110	0.9128	0.9134	0.9138
KIM:	0.9175	0.9166	0.9142	0.9170	0.9170	0.9192	0.9188	0.9221
KIMP:	0.9178	0.9187	0.9096	0.9121	0.9112	0.9118	0.9127	0.9152
PRR%:	0.8354	0.8672	0.8874	0.8909	0.8922	0.8949	0.8946	0.8956
PRRP%:	0.8351	0.8668	0.8886	0.8915	0.8895	0.8930	0.8938	0.8950
KIM%:	0.8259	0.8620	0.8863	0.8877	0.8883	0.8925	0.8908	0.8921
KIMP%:	0.8265	0.8611	0.8836	0.8862	0.8868	0.8893	0.8897	0.8908

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9131	0.9062	0.8978	0.8992	0.9062	0.9091	0.9094	0.9107
PRRP:	0.9088	0.9048	0.8946	0.8953	0.8978	0.9022	0.9020	0.9003
KIM:	0.9151	0.9025	0.8970	0.8968	0.9061	0.9106	0.9106	0.9096
KIMP:	0.9155	0.9066	0.8958	0.8926	0.8998	0.9020	0.9017	0.8992
PRR%:	0.8365	0.8460	0.8684	0.8724	0.8764	0.8741	0.8764	0.8779
PRRP%:	0.8343	0.8443	0.8667	0.8698	0.8744	0.8715	0.8729	0.8755
KIM%:	0.8180	0.8327	0.8598	0.8626	0.8667	0.8663	0.8669	0.8663
KIMP%:	0.8227	0.8312	0.8560	0.8601	0.8634	0.8616	0.8645	0.8644

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8914	0.8984	0.9084	0.9081	0.9147	0.9145	0.9154	0.9162
PRRP:	0.8943	0.8986	0.9058	0.9055	0.9086	0.9089	0.9090	0.9119
KIM:	0.8947	0.8993	0.9079	0.9073	0.9123	0.9151	0.9148	0.9172
KIMP:	0.8952	0.9001	0.9047	0.9071	0.9088	0.9099	0.9081	0.9105
PRR%:	0.8929	0.8966	0.9024	0.9027	0.9071	0.9067	0.9061	0.9082
PRRP%:	0.8933	0.8953	0.9033	0.9030	0.9065	0.9057	0.9064	0.9083
KIM%:	0.8908	0.8919	0.8998	0.8972	0.9012	0.9025	0.9023	0.9027
KIMP%:	0.8919	0.8947	0.8989	0.9017	0.9032	0.9046	0.9028	0.9048

Table B.39. Average Coverage Probability for Model M8 T=200

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.9002	0.9044	0.9123	0.9127	0.9179	0.9170	0.9166	0.9187
PRRP:	0.9014	0.9038	0.9103	0.9099	0.9127	0.9121	0.9121	0.9141
KIM:	0.8996	0.9038	0.9111	0.9145	0.9167	0.9194	0.9178	0.9191
KIMP:	0.8988	0.9025	0.9090	0.9106	0.9117	0.9119	0.9108	0.9114
PRR%:	0.8910	0.8964	0.9002	0.9008	0.9059	0.9035	0.9028	0.9046
PRRP%:	0.8924	0.8950	0.9008	0.9004	0.9057	0.9034	0.9046	0.9069
KIM%:	0.8884	0.8940	0.8977	0.9002	0.9022	0.9030	0.9019	0.9034
KIMP%:	0.8886	0.8957	0.9008	0.9009	0.9042	0.9048	0.9038	0.9034

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9166	0.9173	0.9118	0.9152	0.9155	0.9172	0.9188	0.9207
PRRP:	0.9172	0.9175	0.9089	0.9129	0.9110	0.9124	0.9131	0.9136
KIM:	0.9178	0.9166	0.9126	0.9148	0.9142	0.9166	0.9159	0.9188
KIMP:	0.9178	0.9187	0.9096	0.9121	0.9112	0.9118	0.9127	0.9152
PRR%:	0.8354	0.8672	0.8874	0.8909	0.8922	0.8949	0.8946	0.8956
PRRP%:	0.8347	0.8666	0.8884	0.8912	0.8894	0.8927	0.8936	0.8949
KIM%:	0.8262	0.8620	0.8858	0.8866	0.8870	0.8917	0.8895	0.8917
KIMP%:	0.8265	0.8611	0.8836	0.8862	0.8868	0.8893	0.8897	0.8908

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9275	0.9084	0.9058	0.9087	0.9119	0.9108	0.9152	0.9131
PRRP:	0.9292	0.9106	0.9039	0.9072	0.9054	0.9074	0.9084	0.9083
KIM:	0.9289	0.9070	0.9068	0.9102	0.9093	0.9119	0.9115	0.9153
KIMP:	0.9308	0.9102	0.9026	0.9058	0.9043	0.9048	0.9096	0.9058
PRR%:	0.8124	0.8407	0.8784	0.8806	0.8833	0.8850	0.8867	0.8848
PRRP%:	0.8131	0.8397	0.8749	0.8785	0.8787	0.8825	0.8841	0.8839
KIM%:	0.8001	0.8320	0.8715	0.8742	0.8738	0.8780	0.8772	0.8806
KIMP%:	0.8006	0.8312	0.8685	0.8726	0.8720	0.8765	0.8797	0.8774

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8914	0.8984	0.9084	0.9081	0.9147	0.9145	0.9154	0.9162
PRRP:	0.8943	0.8986	0.9058	0.9055	0.9086	0.9089	0.9090	0.9119
KIM:	0.8947	0.8993	0.9079	0.9073	0.9123	0.9151	0.9148	0.9172
KIMP:	0.8952	0.9001	0.9047	0.9071	0.9088	0.9099	0.9081	0.9105
PRR%:	0.8929	0.8966	0.9024	0.9027	0.9071	0.9067	0.9061	0.9082
PRRP%:	0.8933	0.8953	0.9033	0.9030	0.9065	0.9057	0.9064	0.9083
KIM%:	0.8908	0.8919	0.8998	0.8972	0.9012	0.9025	0.9023	0.9027
KIMP%:	0.8919	0.8947	0.8989	0.9017	0.9032	0.9046	0.9028	0.9048

Table B.40. Average Coverage Probability for Model M9 T=50

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.9079	0.9152	0.9174	0.9185	0.9174	0.9197	0.9196	0.9222
PRRP:	0.8899	0.8885	0.8780	0.8695	0.8614	0.8573	0.8515	0.8466
KIM:	0.8706	0.8753	0.8688	0.8613	0.8550	0.8528	0.8506	0.8505
KIMP:	0.8812	0.8795	0.8741	0.8724	0.8666	0.8671	0.8621	0.8608
PRR%:	0.8926	0.8857	0.8729	0.8598	0.8445	0.8351	0.8260	0.8145
PRRP%:	0.8876	0.8765	0.8628	0.8406	0.8272	0.8136	0.8007	0.7848
KIM%:	0.7908	0.7855	0.7695	0.7540	0.7399	0.7252	0.7109	0.7026
KIMP%:	0.8763	0.8651	0.8503	0.8372	0.8225	0.8105	0.7980	0.7877

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9223	0.9261	0.9229	0.9217	0.9209	0.9214	0.9191	0.9160
PRRP:	0.9009	0.8998	0.8838	0.8803	0.8676	0.8612	0.8522	0.8470
KIM:	0.8951	0.8991	0.8961	0.8869	0.8881	0.8805	0.8757	0.8684
KIMP:	0.9041	0.8987	0.8876	0.8840	0.8787	0.8730	0.8692	0.8608
PRR%:	0.8718	0.8855	0.8769	0.8685	0.8568	0.8441	0.8249	0.8119
PRRP%:	0.8665	0.8762	0.8685	0.8550	0.8380	0.8205	0.8024	0.7866
KIM%:	0.7937	0.7957	0.7795	0.7720	0.7602	0.7459	0.7323	0.7195
KIMP%:	0.8434	0.8506	0.8361	0.8270	0.8113	0.7971	0.7812	0.7674

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9294	0.9277	0.9244	0.9197	0.9204	0.9240	0.9217	0.9230
PRRP:	0.9122	0.9015	0.8909	0.8817	0.8756	0.8666	0.8596	0.8575
KIM:	0.8922	0.8957	0.8906	0.8804	0.8756	0.8659	0.8635	0.8623
KIMP:	0.9137	0.8996	0.8929	0.8844	0.8774	0.8723	0.8668	0.8595
PRR%:	0.8623	0.8799	0.8775	0.8651	0.8558	0.8426	0.8319	0.8184
PRRP%:	0.8548	0.8691	0.8647	0.8523	0.8373	0.8249	0.8108	0.7955
KIM%:	0.7999	0.8058	0.7875	0.7694	0.7524	0.7397	0.7229	0.7113
KIMP%:	0.8331	0.8495	0.8446	0.8360	0.8241	0.8154	0.8026	0.7876

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8933	0.8940	0.8938	0.8915	0.8928	0.8899	0.8949	0.8968
PRRP:	0.8792	0.8676	0.8610	0.8507	0.8468	0.8399	0.8377	0.8331
KIM:	0.8693	0.8610	0.8569	0.8507	0.8512	0.8483	0.8544	0.8528
KIMP:	0.8739	0.8662	0.8616	0.8598	0.8578	0.8536	0.8516	0.8515
PRR%:	0.8921	0.8783	0.8656	0.8538	0.8392	0.8300	0.8208	0.8077
PRRP%:	0.8841	0.8695	0.8496	0.8328	0.8151	0.8007	0.7931	0.7763
KIM%:	0.7912	0.7646	0.7335	0.7094	0.6924	0.6820	0.6723	0.6617
KIMP%:	0.8708	0.8504	0.8311	0.8163	0.8035	0.7914	0.7795	0.7721

Table B.41. Average Coverage Probability for Model M9 T=100

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.9011	0.9077	0.9110	0.9158	0.9156	0.9175	0.9211	0.9222
PRRP:	0.8950	0.8969	0.8940	0.8915	0.8889	0.8855	0.8829	0.8781
KIM:	0.8874	0.8926	0.8903	0.8903	0.8892	0.8881	0.8872	0.8866
KIMP:	0.8934	0.8962	0.8928	0.8917	0.8887	0.8890	0.8844	0.8830
PRR%:	0.8943	0.8950	0.8864	0.8840	0.8764	0.8728	0.8671	0.8636
PRRP%:	0.8929	0.8920	0.8819	0.8756	0.8684	0.8637	0.8560	0.8506
KIM%:	0.8554	0.8554	0.8410	0.8385	0.8274	0.8225	0.8152	0.8100
KIMP%:	0.8889	0.8889	0.8755	0.8739	0.8642	0.8570	0.8497	0.8427

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9124	0.9081	0.9072	0.9094	0.9128	0.9155	0.9190	0.9177
PRRP:	0.9039	0.8937	0.8882	0.8857	0.8871	0.8851	0.8839	0.8800
KIM:	0.8893	0.8810	0.8756	0.8735	0.8779	0.8801	0.8863	0.8875
KIMP:	0.9016	0.8911	0.8881	0.8873	0.8894	0.8869	0.8871	0.8858
PRR%:	0.8450	0.8673	0.8737	0.8745	0.8745	0.8736	0.8734	0.8693
PRRP%:	0.8425	0.8623	0.8665	0.8644	0.8602	0.8592	0.8561	0.8479
KIM%:	0.7957	0.8122	0.8118	0.8139	0.8097	0.8063	0.8075	0.8045
KIMP%:	0.8292	0.8494	0.8550	0.8550	0.8500	0.8481	0.8452	0.8424

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9294	0.9277	0.9244	0.9197	0.9204	0.9240	0.9217	0.9230
PRRP:	0.9122	0.9015	0.8909	0.8817	0.8756	0.8666	0.8596	0.8575
KIM:	0.8922	0.8957	0.8906	0.8804	0.8756	0.8659	0.8635	0.8623
KIMP:	0.9137	0.8996	0.8929	0.8844	0.8774	0.8723	0.8668	0.8595
PRR%:	0.8623	0.8799	0.8775	0.8651	0.8558	0.8426	0.8319	0.8184
PRRP%:	0.8548	0.8691	0.8647	0.8523	0.8373	0.8249	0.8108	0.7955
KIM%:	0.7999	0.8058	0.7875	0.7694	0.7524	0.7397	0.7229	0.7113
KIMP%:	0.8331	0.8495	0.8446	0.8360	0.8241	0.8154	0.8026	0.7876

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8962	0.9012	0.9026	0.9046	0.9062	0.9102	0.9098	0.9125
PRRP:	0.8912	0.8911	0.8866	0.8844	0.8817	0.8765	0.8770	0.8721
KIM:	0.8867	0.8900	0.8867	0.8823	0.8827	0.8775	0.8794	0.8767
KIMP:	0.8913	0.8909	0.8907	0.8879	0.8847	0.8800	0.8801	0.8780
PRR%:	0.8934	0.8924	0.8861	0.8820	0.8767	0.8742	0.8672	0.8637
PRRP%:	0.8923	0.8896	0.8833	0.8791	0.8726	0.8654	0.8613	0.8548
KIM%:	0.8528	0.8449	0.8322	0.8200	0.8163	0.8080	0.8030	0.7973
KIMP%:	0.8879	0.8828	0.8759	0.8667	0.8614	0.8525	0.8481	0.8411

Table B.42. Average Coverage Probability for Model M9 T=200

Nor \ h:	1	2	3	4	5	6	7	8
PRR:	0.9011	0.9077	0.9110	0.9158	0.9156	0.9175	0.9211	0.9222
PRRP:	0.8950	0.8969	0.8940	0.8915	0.8889	0.8855	0.8829	0.8781
KIM:	0.8874	0.8926	0.8903	0.8903	0.8892	0.8881	0.8872	0.8866
KIMP:	0.8934	0.8962	0.8928	0.8917	0.8887	0.8890	0.8844	0.8830
PRR%:	0.8943	0.8950	0.8864	0.8840	0.8764	0.8728	0.8671	0.8636
PRRP%:	0.8929	0.8920	0.8819	0.8756	0.8684	0.8637	0.8560	0.8506
KIM%:	0.8554	0.8554	0.8410	0.8385	0.8274	0.8225	0.8152	0.8100
KIMP%:	0.8889	0.8889	0.8755	0.8739	0.8642	0.8570	0.8497	0.8427

χ^2 \ h:	1	2	3	4	5	6	7	8
PRR:	0.9124	0.9081	0.9072	0.9094	0.9128	0.9155	0.9190	0.9177
PRRP:	0.9034	0.8949	0.8883	0.8867	0.8885	0.8865	0.8858	0.8821
KIM:	0.8903	0.8804	0.8730	0.8701	0.8735	0.8743	0.8798	0.8797
KIMP:	0.9016	0.8911	0.8881	0.8873	0.8894	0.8869	0.8871	0.8858
PRR%:	0.8450	0.8673	0.8737	0.8745	0.8745	0.8736	0.8734	0.8693
PRRP%:	0.8424	0.8631	0.8684	0.8672	0.8644	0.8635	0.8618	0.8551
KIM%:	0.7974	0.8140	0.8135	0.8132	0.8102	0.8065	0.8088	0.8076
KIMP%:	0.8292	0.8494	0.8550	0.8550	0.8500	0.8481	0.8452	0.8424

Exp \ h:	1	2	3	4	5	6	7	8
PRR:	0.9318	0.9252	0.9187	0.9157	0.9138	0.9120	0.9143	0.9138
PRRP:	0.9241	0.9097	0.8983	0.8908	0.8837	0.8794	0.8717	0.8674
KIM:	0.9103	0.8955	0.8922	0.8846	0.8809	0.8802	0.8770	0.8762
KIMP:	0.9198	0.9062	0.8986	0.8898	0.8892	0.8868	0.8810	0.8773
PRR%:	0.8373	0.8578	0.8658	0.8690	0.8681	0.8641	0.8641	0.8655
PRRP%:	0.8316	0.8490	0.8569	0.8563	0.8544	0.8540	0.8529	0.8468
KIM%:	0.7982	0.8103	0.8081	0.8034	0.7997	0.8002	0.7990	0.7985
KIMP%:	0.8178	0.8367	0.8419	0.8424	0.8398	0.8406	0.8353	0.8304

t-dist \ h:	1	2	3	4	5	6	7	8
PRR:	0.8962	0.9012	0.9026	0.9046	0.9062	0.9102	0.9098	0.9125
PRRP:	0.8912	0.8911	0.8866	0.8844	0.8817	0.8765	0.8770	0.8721
KIM:	0.8867	0.8900	0.8867	0.8823	0.8827	0.8775	0.8794	0.8767
KIMP:	0.8913	0.8909	0.8907	0.8879	0.8847	0.8800	0.8801	0.8780
PRR%:	0.8934	0.8924	0.8861	0.8820	0.8767	0.8742	0.8672	0.8637
PRRP%:	0.8923	0.8896	0.8833	0.8791	0.8726	0.8654	0.8613	0.8548
KIM%:	0.8528	0.8449	0.8322	0.8200	0.8163	0.8080	0.8030	0.7973
KIMP%:	0.8879	0.8828	0.8759	0.8667	0.8614	0.8525	0.8481	0.8411

Table B.43. Variance and Lengths of Intervals for Model M6 T=50

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	3.029	3.935	4.213	3.811	2.874	2.858	4.611	3.359
Var	3	2.858	3.471	2.700	3.191	3.535	3.530	4.611	4.606
	8	2.742	2.828	2.570	2.790	3.498	3.482	4.730	4.094
	1	4.200	4.228	4.201	4.199	4.066	4.122	3.957	4.080
Len1	3	6.568	6.589	6.585	6.702	6.041	6.054	5.821	6.036
	8	7.609	7.284	7.566	7.474	6.303	6.231	5.986	6.226
	1	4.102	4.105	4.112	4.123	3.979	3.985	3.942	4.011
Len2	3	5.883	5.867	5.862	5.827	5.570	5.601	5.525	5.593
	8	6.331	6.384	6.300	6.290	5.711	5.720	5.654	5.772
Chi	1	4.679	5.747	5.623	6.076	3.292	3.156	4.945	4.063
Var	3	1.951	2.159	4.840	2.650	4.583	6.020	5.964	7.161
	8	1.679	2.230	2.162	2.067	3.944	5.224	6.484	6.706
	1	4.282	4.278	4.263	4.247	4.261	4.276	4.049	4.137
Len1	3	6.567	6.656	6.526	6.707	6.231	6.256	5.848	6.066
	8	7.722	7.461	7.684	7.615	6.594	6.600	6.141	6.333
	1	4.044	4.049	4.061	4.100	4.006	4.006	3.907	3.978
Len2	3	6.072	6.039	6.026	6.057	5.831	5.826	5.688	5.812
	8	6.504	6.526	6.480	6.454	5.919	5.958	5.827	5.896
Exp	1	4.228	2.926	4.743	2.214	4.520	5.176	4.159	4.867
Var	3	2.611	2.511	3.265	2.479	3.783	4.123	4.518	4.482
	8	2.637	3.248	2.782	2.834	3.759	3.886	4.746	4.497
	1	4.170	4.145	4.180	4.102	4.174	4.187	3.927	4.012
Len1	3	6.240	6.277	6.190	6.328	6.017	5.996	5.525	5.760
	8	7.152	6.892	7.121	6.985	6.251	6.174	5.773	5.931
	1	4.130	4.230	4.181	4.277	4.205	4.296	4.061	4.141
Len2	3	6.218	6.236	6.214	6.163	6.030	6.110	5.877	5.945
	8	6.660	6.586	6.548	6.566	6.159	6.141	5.932	6.018
t-dist	1	2.721	2.478	6.309	2.559	2.126	2.405	3.228	2.418
Var	3	2.319	2.847	2.376	2.643	2.698	3.105	3.346	3.292
	8	2.041	2.742	1.983	2.327	3.266	2.755	3.674	3.647
	1	4.204	4.225	4.218	4.291	4.197	4.200	3.990	4.185
Len1	3	6.709	6.743	6.677	6.819	6.308	6.334	5.990	6.157
	8	7.745	7.508	7.649	7.587	6.593	6.596	6.159	6.391
	1	4.342	4.391	4.336	4.413	4.261	4.282	4.194	4.289
Len2	3	6.063	6.011	5.935	5.997	5.820	5.828	5.674	5.779
	8	6.505	6.487	6.437	6.440	5.957	5.928	5.833	5.869

Table B.44. Variance and Lengths of Intervals for Model M7 T=50

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	2.864	2.438	2.675	2.277	3.051	2.815	4.385	3.622
Var	3	2.311	2.140	1.898	1.873	3.514	3.118	5.098	4.120
	8	2.113	2.350	1.978	1.923	3.618	3.902	5.344	4.328
	1	4.166	4.166	4.136	4.165	4.033	4.057	3.888	4.016
Len1	3	6.478	6.507	6.529	6.620	5.966	5.969	5.739	5.954
	8	7.302	7.095	7.315	7.211	6.134	6.108	5.838	6.033
	1	4.127	4.170	4.159	4.168	4.000	4.042	3.994	4.041
Len2	3	5.503	5.466	5.451	5.466	5.229	5.231	5.136	5.227
	8	5.761	5.744	5.698	5.766	5.332	5.311	5.208	5.322
Chi	1	2.094	1.821	3.296	2.677	2.639	2.850	3.826	3.387
Var	3	1.870	1.930	4.891	2.532	3.250	3.030	3.671	3.129
	8	1.995	2.074	5.832	2.780	3.537	3.155	3.884	3.324
	1	4.289	4.307	4.300	4.303	4.249	4.291	4.094	4.185
Len1	3	6.628	6.654	6.517	6.657	6.272	6.280	5.870	6.044
	8	7.648	7.377	7.491	7.417	6.534	6.514	6.095	6.249
	1	4.097	4.128	4.131	4.168	4.056	4.075	4.005	4.070
Len2	3	5.566	5.512	5.496	5.486	5.328	5.299	5.209	5.266
	8	5.737	5.745	5.740	5.725	5.372	5.373	5.300	5.345
Exp	1	7.227	6.159	13.231	6.015	3.020	2.917	3.993	3.597
Var	3	2.050	2.127	5.955	2.650	4.624	5.129	6.317	5.150
	8	1.986	2.226	2.262	2.162	3.681	3.835	6.625	4.777
	1	4.221	4.182	4.233	4.245	4.303	4.287	3.990	4.150
Len1	3	6.444	6.483	6.442	6.571	6.292	6.263	5.677	5.951
	8	7.472	7.160	7.432	7.339	6.565	6.503	5.879	6.130
	1	4.160	4.103	4.168	4.149	4.168	4.126	4.029	4.063
Len2	3	5.556	5.516	5.516	5.539	5.399	5.389	5.257	5.342
	8	5.796	5.772	5.680	5.695	5.483	5.486	5.236	5.346
t-dist	1	2.094	2.085	2.186	1.843	2.417	2.895	3.869	2.653
Var	3	2.891	2.716	2.553	2.531	4.253	4.639	5.617	4.907
	8	2.843	2.769	2.531	2.654	4.662	4.580	6.235	5.620
	1	4.285	4.391	4.378	4.265	4.230	4.298	4.163	4.140
Len1	3	6.584	6.607	6.626	6.668	6.152	6.203	5.960	6.037
	8	7.501	7.278	7.496	7.289	6.405	6.399	6.102	6.152
	1	4.318	4.231	4.269	4.250	4.232	4.180	4.105	4.123
Len2	3	5.483	5.523	5.476	5.407	5.248	5.318	5.152	5.178
	8	5.687	5.682	5.679	5.687	5.344	5.330	5.214	5.297

Table B.45. Variance and Lengths of Intervals for Model M8 T=50

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	3.353	3.109	4.119	2.998	3.408	3.596	3.800	3.375
Var	3	2.068	2.403	2.375	2.266	3.467	3.597	3.373	3.663
	8	1.317	1.422	1.146	1.349	1.996	1.874	2.445	2.448
	1	4.113	4.118	4.107	4.149	3.991	4.003	3.917	4.017
Len1	3	4.700	4.607	4.696	4.606	4.460	4.479	4.388	4.483
	8	5.117	4.868	5.162	4.858	4.692	4.695	4.651	4.688
	1	4.121	4.143	4.179	4.157	3.999	4.019	4.006	4.026
Len2	3	6.111	5.967	6.127	6.041	5.761	5.739	5.661	5.773
	8	6.565	6.355	6.649	6.350	5.999	6.046	5.955	6.007
Chi	1	4.408	5.060	4.700	4.613	5.650	6.756	5.656	5.178
Var	3	2.617	1.915	3.129	1.940	3.676	7.793	2.770	6.798
	8	1.465	1.426	1.274	1.460	1.980	2.118	2.488	2.355
	1	3.976	4.038	4.027	4.008	3.949	3.997	3.860	3.896
Len1	3	4.623	4.548	4.703	4.537	4.493	4.510	4.389	4.399
	8	4.921	4.750	4.996	4.777	4.616	4.659	4.492	4.587
	1	4.120	4.130	4.154	4.170	4.094	4.103	4.051	4.076
Len2	3	6.279	6.243	6.323	6.220	5.999	6.015	5.948	5.995
	8	6.710	6.512	6.760	6.485	6.212	6.236	6.115	6.157
Exp	1	3.790	4.992	4.075	3.437	3.987	4.281	4.707	4.472
Var	3	4.286	6.710	7.811	4.848	4.878	4.074	4.851	4.866
	8	2.514	2.662	3.188	2.678	3.249	3.039	3.503	3.349
	1	4.090	4.094	4.203	4.152	4.153	4.183	4.022	4.046
Len1	3	4.714	4.635	4.816	4.649	4.655	4.638	4.507	4.487
	8	5.044	4.830	5.102	4.900	4.794	4.774	4.602	4.667
	1	4.057	4.100	4.131	4.137	4.068	4.083	4.029	4.039
Len2	3	6.381	6.312	6.462	6.340	6.111	6.166	6.041	6.105
	8	6.799	6.563	6.797	6.485	6.327	6.315	6.154	6.187
t-dist	1	2.730	2.588	2.794	2.679	2.598	2.977	2.973	3.608
Var	3	2.120	2.328	2.458	2.264	2.940	2.957	3.560	3.514
	8	1.713	1.990	1.682	1.811	2.718	2.779	3.120	2.826
	1	4.241	4.259	4.236	4.234	4.196	4.213	4.020	4.091
Len1	3	4.801	4.724	4.813	4.788	4.646	4.696	4.481	4.615
	8	5.230	5.059	5.232	4.963	4.898	4.950	4.671	4.759
	1	4.120	4.207	4.158	4.219	4.068	4.126	4.036	4.095
Len2	3	6.201	6.134	6.220	6.123	5.944	5.957	5.818	5.868
	8	6.663	6.475	6.681	6.394	6.190	6.226	6.019	6.061

Table B.46. Variance and Lengths of Intervals for Model M9 T=50

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	3.151	3.814	6.173	4.035	3.980	3.620	15.293	3.896
Var	3	7.108	9.736	15.641	9.292	9.574	8.214	22.015	8.157
	8	9.180	16.814	25.739	16.366	21.539	19.299	33.203	18.963
	1	4.591	4.462	4.281	4.216	4.490	4.398	3.694	4.125
Len1	3	6.268	5.661	6.051	5.366	5.890	5.543	4.683	5.065
	8	8.981	7.094	9.407	7.122	7.411	6.792	5.882	6.203
	1	4.361	4.289	4.282	4.183	4.240	4.189	3.608	4.052
Len2	3	11.869	11.289	11.629	11.085	11.160	10.937	9.447	10.378
	8	22.425	19.603	21.885	20.156	19.596	18.943	15.984	17.671
Chi	1	2.355	6.149	6.551	4.791	3.882	3.446	15.282	5.048
Var	3	4.066	8.241	6.559	8.287	8.505	7.270	27.650	10.476
	8	9.435	18.147	20.350	17.303	24.103	23.242	44.351	29.373
	1	4.499	4.404	4.343	4.197	4.492	4.433	3.731	4.077
Len1	3	6.004	5.533	6.082	5.303	5.744	5.537	4.706	4.963
	8	8.290	6.760	9.225	6.913	6.957	6.441	5.746	5.922
	1	4.437	4.423	4.490	4.350	4.391	4.413	3.694	4.227
Len2	3	11.806	11.397	12.240	11.166	11.398	11.206	9.542	10.481
	8	21.402	18.869	22.457	19.764	19.024	18.516	15.706	17.003
Exp	1	3.851	6.555	14.922	5.291	4.241	4.160	11.951	6.386
Var	3	7.078	10.129	14.204	11.193	10.121	8.753	24.734	10.860
	8	10.515	14.996	27.454	23.086	23.730	25.252	43.812	24.425
	1	4.557	4.518	4.346	4.196	4.610	4.609	3.810	4.137
Len1	3	6.057	5.574	6.238	5.334	5.905	5.665	4.844	5.072
	8	8.475	6.935	9.905	7.009	7.256	6.770	5.969	6.113
	1	4.509	4.364	4.430	4.259	4.477	4.404	3.741	4.167
Len2	3	12.166	11.633	12.327	11.469	11.889	11.578	10.040	10.745
	8	23.078	20.232	23.657	21.340	20.428	20.007	16.909	18.384
t-dist	1	4.843	5.451	5.693	4.459	3.310	3.223	18.952	4.025
Var	3	11.258	13.634	16.439	13.275	12.464	12.288	36.489	14.100
	8	20.570	23.530	22.563	20.510	26.243	25.272	48.289	26.625
	1	4.662	4.450	4.378	4.258	4.599	4.481	3.738	4.154
Len1	3	6.219	5.683	6.054	5.438	5.920	5.614	4.588	5.066
	8	8.479	6.962	8.877	7.058	7.173	6.717	5.489	6.065
	1	4.492	4.456	4.419	4.333	4.412	4.386	3.663	4.194
Len2	3	11.718	11.216	11.376	10.961	11.187	10.907	9.101	10.241
	8	21.386	18.988	20.896	19.656	18.928	18.705	15.239	17.187

Table B.47. Variance and Lengths of Intervals for Model M6 T=100

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	1.249	1.121	1.504	1.104	1.521	1.494	2.088	1.718
Var	3	0.992	0.948	0.969	0.845	1.176	1.325	1.630	1.281
	8	1.027	1.013	1.092	0.965	1.412	1.299	1.543	1.406
	1	4.046	4.056	4.025	4.041	3.999	4.004	3.932	3.993
Len1	3	6.199	6.189	6.185	6.233	5.958	5.915	5.857	5.938
	8	6.792	6.678	6.789	6.757	6.230	6.214	6.153	6.254
	1	3.962	3.962	3.970	3.978	3.905	3.920	3.901	3.913
Len2	3	5.645	5.635	5.640	5.611	5.514	5.489	5.503	5.482
	8	5.887	5.880	5.864	5.847	5.607	5.613	5.579	5.600
Chi	1	2.207	2.034	2.851	2.081	1.543	1.620	1.808	1.723
Var	3	1.718	1.643	1.917	1.538	2.112	2.189	2.340	2.093
	8	1.494	1.547	1.539	1.448	1.929	2.070	2.501	2.108
	1	3.970	3.983	4.000	4.002	3.975	4.013	3.875	3.951
Len1	3	6.023	6.031	6.046	6.089	5.882	5.861	5.686	5.808
	8	6.548	6.475	6.533	6.503	6.111	6.080	5.866	6.018
	1	3.971	4.007	3.992	3.980	3.959	3.991	3.920	3.931
Len2	3	5.814	5.814	5.778	5.809	5.716	5.711	5.592	5.683
	8	6.027	6.038	5.996	6.095	5.802	5.816	5.693	5.832
Exp	1	2.928	2.478	6.361	2.132	2.063	2.370	1.998	2.371
Var	3	1.078	1.138	1.676	1.160	1.727	1.994	2.099	2.217
	8	1.019	1.097	1.065	1.090	1.640	1.689	2.146	1.966
	1	4.049	4.040	4.018	4.056	4.128	4.118	3.891	4.008
Len1	3	6.117	6.116	6.090	6.122	6.043	6.033	5.707	5.813
	8	6.647	6.591	6.640	6.590	6.288	6.287	5.919	6.082
	1	4.046	4.035	4.040	4.071	4.080	4.074	3.962	4.022
Len2	3	6.077	6.062	6.067	6.054	5.987	6.013	5.889	5.927
	8	6.325	6.358	6.298	6.311	6.100	6.180	5.979	6.073
t-dist	1	0.906	0.908	1.126	0.875	0.887	0.921	1.075	0.922
Var	3	1.198	1.142	1.205	1.396	1.369	1.292	1.485	1.537
	8	1.394	1.531	1.415	1.519	1.641	1.598	1.823	1.850
	1	4.180	4.187	4.173	4.200	4.187	4.190	4.068	4.159
Len1	3	6.290	6.340	6.304	6.375	6.124	6.151	5.979	6.074
	8	6.819	6.827	6.841	6.741	6.363	6.416	6.220	6.267
	1	4.119	4.066	4.074	4.104	4.098	4.073	4.011	4.051
Len2	3	5.778	5.790	5.758	5.781	5.682	5.713	5.606	5.660
	8	6.083	6.031	6.020	6.018	5.891	5.813	5.746	5.776

Table B.48. Variance and Lengths of Intervals for Model M7 T=100

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	1.341	1.115	1.288	1.331	1.528	1.400	1.646	1.673
Var	3	0.888	0.957	1.191	1.009	1.437	1.382	1.673	1.493
	8	1.021	1.010	1.064	0.980	1.465	1.184	1.561	1.401
	1	4.059	4.039	4.039	4.035	4.003	3.987	3.940	3.979
Len1	3	6.139	6.139	6.119	6.156	5.872	5.870	5.787	5.878
	8	6.613	6.557	6.645	6.608	6.068	6.089	6.021	6.075
	1	4.091	4.100	4.128	4.097	4.029	4.053	4.057	4.040
Len2	3	5.314	5.307	5.261	5.289	5.178	5.199	5.130	5.192
	8	5.443	5.444	5.418	5.453	5.249	5.250	5.224	5.275
Chi	1	2.078	1.808	2.101	1.699	1.677	1.776	2.073	1.927
Var	3	2.033	1.954	1.996	1.777	2.266	2.488	2.985	2.444
	8	2.038	2.141	1.833	1.894	3.027	2.499	3.508	2.960
	1	3.871	3.866	3.897	3.902	3.886	3.882	3.795	3.857
Len1	3	5.969	5.941	6.033	5.994	5.821	5.801	5.666	5.731
	8	6.543	6.385	6.574	6.490	6.069	6.031	5.899	5.979
	1	4.020	3.941	4.016	3.970	4.009	3.946	3.950	3.933
Len2	3	5.290	5.344	5.267	5.286	5.202	5.256	5.137	5.184
	8	5.390	5.428	5.418	5.420	5.269	5.275	5.220	5.230
Exp	1	2.406	2.346	10.849	2.319	1.960	1.906	2.100	2.197
Var	3	1.877	1.922	3.806	1.751	2.417	2.190	2.598	2.545
	8	1.972	1.711	1.648	1.661	2.546	2.528	3.069	2.902
	1	3.950	4.003	3.976	3.978	4.030	4.069	3.894	3.936
Len1	3	6.091	6.046	6.117	6.118	6.010	5.975	5.785	5.840
	8	6.661	6.518	6.675	6.525	6.308	6.275	5.992	6.048
	1	4.037	3.946	4.003	3.992	4.061	3.998	3.943	3.964
Len2	3	5.409	5.392	5.398	5.400	5.350	5.341	5.251	5.268
	8	5.525	5.517	5.521	5.536	5.429	5.404	5.312	5.351
t-dist	1	1.318	1.065	1.372	1.090	1.089	1.009	1.160	1.050
Var	3	1.560	1.577	1.476	1.519	1.986	1.941	2.324	2.132
	8	1.545	1.601	1.513	1.669	2.095	2.235	2.492	2.154
	1	4.191	4.147	4.192	4.216	4.179	4.159	4.074	4.155
Len1	3	6.282	6.312	6.260	6.349	6.085	6.132	5.921	6.050
	8	6.809	6.741	6.805	6.732	6.301	6.322	6.136	6.243
	1	4.042	4.066	4.029	4.057	4.034	4.051	3.976	4.014
Len2	3	5.350	5.364	5.325	5.358	5.259	5.275	5.212	5.277
	8	5.505	5.518	5.444	5.482	5.345	5.357	5.262	5.302

Table B.49. Variance and Lengths of Intervals for Model M8 T=100

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	1.203	1.238	1.289	1.096	1.397	1.405	1.599	1.508
Var	3	1.210	1.165	1.211	1.177	1.364	1.385	1.472	1.396
	8	1.098	1.266	1.135	1.210	1.350	1.394	1.499	1.480
	1	4.064	4.061	4.068	4.030	4.005	4.005	3.977	3.982
Len1	3	4.584	4.538	4.602	4.522	4.489	4.481	4.456	4.486
	8	4.843	4.774	4.848	4.737	4.672	4.706	4.639	4.674
	1	4.034	4.056	4.022	4.010	3.969	3.997	3.967	3.957
Len2	3	6.006	5.996	5.979	5.978	5.834	5.858	5.804	5.861
	8	6.321	6.243	6.343	6.218	6.061	6.114	6.067	6.078
Chi	1	2.057	2.197	1.898	1.974	1.839	2.048	2.069	2.141
Var	3	1.462	1.393	1.225	1.303	1.478	1.467	1.606	1.529
	8	0.990	1.092	1.076	1.122	1.174	1.209	1.320	1.314
	1	4.013	4.073	4.003	4.023	4.030	4.087	3.950	3.986
Len1	3	4.689	4.599	4.697	4.607	4.630	4.616	4.559	4.574
	8	4.891	4.775	4.968	4.820	4.749	4.768	4.747	4.750
	1	4.103	4.131	4.101	4.113	4.102	4.135	4.043	4.072
Len2	3	6.309	6.275	6.311	6.275	6.171	6.221	6.142	6.174
	8	6.607	6.492	6.604	6.482	6.378	6.386	6.322	6.353
Exp	1	1.564	1.425	1.820	1.657	1.698	1.624	1.762	1.811
Var	3	1.339	1.256	1.382	1.271	1.771	1.672	1.819	1.736
	8	1.398	1.535	1.404	1.573	1.733	1.914	1.937	1.947
	1	3.951	3.931	3.978	3.928	4.022	4.002	3.888	3.882
Len1	3	4.720	4.593	4.715	4.578	4.712	4.661	4.563	4.541
	8	4.903	4.791	4.914	4.756	4.816	4.848	4.690	4.688
	1	3.952	3.938	4.056	3.995	3.995	4.016	4.007	3.962
Len2	3	6.308	6.249	6.270	6.227	6.210	6.182	6.081	6.132
	8	6.568	6.457	6.660	6.446	6.379	6.381	6.343	6.362
t-dist	1	1.077	1.039	1.141	1.067	1.157	1.122	1.251	1.173
Var	3	0.967	1.097	0.954	0.955	1.140	1.164	1.219	1.157
	8	1.009	1.059	1.038	1.117	1.229	1.236	1.453	1.274
	1	4.042	4.127	4.111	4.118	4.025	4.103	4.017	4.049
Len1	3	4.611	4.556	4.638	4.546	4.554	4.562	4.519	4.513
	8	4.876	4.825	4.885	4.780	4.753	4.803	4.661	4.708
	1	4.120	4.092	4.160	4.141	4.116	4.082	4.104	4.078
Len2	3	6.077	6.064	6.098	6.017	5.951	5.987	5.896	5.914
	8	6.443	6.329	6.422	6.255	6.223	6.218	6.138	6.138

Table B.50. Variance and Lengths of Intervals for Model M9 T=100

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	0.980	0.957	1.740	1.250	1.346	1.123	4.221	1.571
Var	3	1.401	1.806	3.368	2.025	2.286	2.436	6.185	2.690
	8	2.832	5.542	10.236	5.407	5.982	5.967	13.566	6.895
	1	4.114	4.082	4.018	4.054	4.060	4.040	3.764	3.984
Len1	3	5.287	5.081	5.189	5.013	5.073	4.995	4.573	4.852
	8	7.122	6.321	7.051	6.273	6.336	6.104	5.529	5.790
	1	4.083	4.072	4.066	4.042	4.034	4.016	3.757	3.989
Len2	3	10.444	10.259	10.287	10.296	10.049	9.960	9.292	9.925
	8	18.181	17.100	17.722	17.454	16.825	16.475	15.370	16.383
Chi	1	2.716	3.315	5.433	3.110	2.443	2.245	6.748	2.534
Var	3	3.768	4.993	10.046	4.207	4.003	4.025	9.218	4.452
	8	3.558	6.269	6.215	4.888	6.351	8.941	10.639	8.805
	1	4.110	4.066	3.987	3.971	4.127	4.095	3.662	3.931
Len1	3	5.409	5.173	5.351	5.093	5.232	5.168	4.579	4.941
	8	7.376	6.513	7.622	6.528	6.645	6.332	5.655	6.046
	1	3.985	3.974	3.971	3.976	3.992	3.979	3.581	3.960
Len2	3	10.359	10.120	10.144	10.205	10.103	9.999	8.888	9.900
	8	18.140	17.147	17.712	17.409	17.004	16.726	15.065	16.402
Exp	1	0.996	2.285	7.274	2.246	2.737	2.574	4.092	2.795
Var	3	2.562	3.595	5.230	2.923	5.011	4.760	11.364	5.879
	8	5.124	8.549	7.949	5.722	9.883	9.296	20.713	12.409
	1	4.088	4.125	3.981	3.980	4.184	4.204	3.694	3.946
Len1	3	5.476	5.196	5.320	5.066	5.420	5.270	4.611	4.932
	8	7.396	6.551	7.256	6.443	6.837	6.519	5.601	5.995
	1	4.060	4.056	4.066	3.941	4.096	4.088	3.711	3.922
Len2	3	10.355	10.209	10.243	10.195	10.248	10.145	9.122	9.942
	8	18.025	17.002	17.440	17.177	17.044	16.756	15.160	16.223
t-dist	1	1.462	1.606	1.768	1.679	1.956	1.712	6.655	2.154
Var	3	2.234	2.826	3.221	2.253	3.585	3.535	10.424	4.313
	8	3.335	5.753	7.508	4.320	6.641	5.929	15.805	9.231
	1	4.144	4.100	4.057	4.112	4.131	4.113	3.769	4.050
Len1	3	5.382	5.135	5.266	5.093	5.194	5.107	4.590	4.940
	8	7.147	6.409	7.091	6.343	6.402	6.177	5.451	5.881
	1	4.198	4.214	4.170	4.185	4.183	4.209	3.801	4.134
Len2	3	10.768	10.604	10.500	10.668	10.516	10.506	9.423	10.343
	8	18.487	17.629	17.727	17.692	17.233	17.149	15.261	16.633

Table B.51. Variance and Lengths of Intervals for Model M6 T=200

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	1.249	1.121	1.504	1.104	1.521	1.494	2.088	1.718
Var	3	0.992	0.948	0.969	0.845	1.176	1.325	1.630	1.281
	8	1.027	1.013	1.092	0.965	1.412	1.299	1.543	1.406
	1	4.046	4.056	4.025	4.041	3.999	4.004	3.932	3.993
Len1	3	6.199	6.189	6.185	6.233	5.958	5.915	5.857	5.938
	8	6.792	6.678	6.789	6.757	6.230	6.214	6.153	6.254
	1	3.962	3.962	3.970	3.978	3.905	3.920	3.901	3.913
Len2	3	5.645	5.635	5.640	5.611	5.514	5.489	5.503	5.482
	8	5.887	5.880	5.864	5.847	5.607	5.613	5.579	5.600
Chi	1	2.207	2.015	2.711	2.081	1.543	1.621	1.753	1.723
Var	3	1.718	1.671	1.876	1.538	2.112	2.169	2.285	2.093
	8	1.494	1.562	1.575	1.448	1.929	2.046	2.403	2.108
	1	3.970	3.982	4.003	4.002	3.975	4.011	3.880	3.951
Len1	3	6.023	6.030	6.055	6.089	5.882	5.862	5.692	5.808
	8	6.548	6.483	6.497	6.503	6.111	6.077	5.864	6.018
	1	3.971	4.006	3.993	3.980	3.959	3.988	3.921	3.931
Len2	3	5.814	5.815	5.772	5.809	5.716	5.709	5.594	5.683
	8	6.027	6.039	5.994	6.095	5.802	5.814	5.692	5.832
Exp	1	2.928	2.478	6.361	2.132	2.063	2.370	1.998	2.371
Var	3	1.078	1.138	1.676	1.160	1.727	1.994	2.099	2.217
	8	1.019	1.097	1.065	1.090	1.640	1.689	2.146	1.966
	1	4.049	4.040	4.018	4.056	4.128	4.118	3.891	4.008
Len1	3	6.117	6.116	6.090	6.122	6.043	6.033	5.707	5.813
	8	6.647	6.591	6.640	6.590	6.288	6.287	5.919	6.082
	1	4.046	4.035	4.040	4.071	4.080	4.074	3.962	4.022
Len2	3	6.077	6.062	6.067	6.054	5.987	6.013	5.889	5.927
	8	6.325	6.358	6.298	6.311	6.100	6.180	5.979	6.073
t-dist	1	0.906	0.908	1.126	0.875	0.887	0.921	1.075	0.922
Var	3	1.198	1.142	1.205	1.396	1.369	1.292	1.485	1.537
	8	1.394	1.531	1.415	1.519	1.641	1.598	1.823	1.850
	1	4.180	4.187	4.173	4.200	4.187	4.190	4.068	4.159
Len1	3	6.290	6.340	6.304	6.375	6.124	6.151	5.979	6.074
	8	6.819	6.827	6.841	6.741	6.363	6.416	6.220	6.267
	1	4.119	4.066	4.074	4.104	4.098	4.073	4.011	4.051
Len2	3	5.778	5.790	5.758	5.781	5.682	5.713	5.606	5.660
	8	6.083	6.031	6.020	6.018	5.891	5.813	5.746	5.776

Table B.52. Variance and Lengths of Intervals for Model M7 T=200

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	1.341	1.115	1.288	1.331	1.528	1.400	1.646	1.673
Var	3	0.888	0.957	1.191	1.009	1.437	1.382	1.673	1.493
	8	1.021	1.010	1.064	0.980	1.465	1.184	1.561	1.401
	1	4.059	4.039	4.039	4.035	4.003	3.987	3.940	3.979
Len1	3	6.139	6.139	6.119	6.156	5.872	5.870	5.787	5.878
	8	6.613	6.557	6.645	6.608	6.068	6.089	6.021	6.075
	1	4.091	4.100	4.128	4.097	4.029	4.053	4.057	4.040
Len2	3	5.314	5.307	5.261	5.289	5.178	5.199	5.130	5.192
	8	5.443	5.444	5.418	5.453	5.249	5.250	5.224	5.275
Chi	1	2.078	1.809	1.886	1.699	1.677	1.775	2.025	1.927
Var	3	2.033	1.951	1.945	1.777	2.266	2.463	2.864	2.444
	8	2.038	2.111	1.962	1.894	3.027	2.493	3.361	2.960
	1	3.871	3.865	3.899	3.902	3.886	3.879	3.800	3.857
Len1	3	5.969	5.940	6.032	5.994	5.821	5.800	5.670	5.731
	8	6.543	6.390	6.514	6.490	6.069	6.032	5.896	5.979
	1	4.020	3.941	4.016	3.970	4.009	3.945	3.950	3.933
Len2	3	5.290	5.345	5.264	5.286	5.202	5.254	5.139	5.184
	8	5.390	5.427	5.420	5.420	5.269	5.273	5.220	5.230
Exp	1	2.406	2.346	10.849	2.319	1.960	1.906	2.100	2.197
Var	3	1.877	1.922	3.806	1.751	2.417	2.190	2.598	2.545
	8	1.972	1.711	1.648	1.661	2.546	2.528	3.069	2.902
	1	3.950	4.003	3.976	3.978	4.030	4.069	3.894	3.936
Len1	3	6.091	6.046	6.117	6.118	6.010	5.975	5.785	5.840
	8	6.661	6.518	6.675	6.525	6.308	6.275	5.992	6.048
	1	4.037	3.946	4.003	3.992	4.061	3.998	3.943	3.964
Len2	3	5.409	5.392	5.398	5.400	5.350	5.341	5.251	5.268
	8	5.525	5.517	5.521	5.536	5.429	5.404	5.312	5.351
t-dist	1	1.318	1.065	1.372	1.090	1.089	1.009	1.160	1.050
Var	3	1.560	1.577	1.476	1.519	1.986	1.941	2.324	2.132
	8	1.545	1.601	1.513	1.669	2.095	2.235	2.492	2.154
	1	4.191	4.147	4.192	4.216	4.179	4.159	4.074	4.155
Len1	3	6.282	6.312	6.260	6.349	6.085	6.132	5.921	6.050
	8	6.809	6.741	6.805	6.732	6.301	6.322	6.136	6.243
	1	4.042	4.066	4.029	4.057	4.034	4.051	3.976	4.014
Len2	3	5.350	5.364	5.325	5.358	5.259	5.275	5.212	5.277
	8	5.505	5.518	5.444	5.482	5.345	5.357	5.262	5.302

Table B.53. Variance and Lengths of Intervals for Model M8 T=200

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	1.203	1.238	1.289	1.096	1.397	1.405	1.599	1.508
Var	3	1.210	1.165	1.211	1.177	1.364	1.385	1.472	1.396
	8	1.098	1.266	1.135	1.210	1.350	1.394	1.499	1.480
	1	4.064	4.061	4.068	4.030	4.005	4.005	3.977	3.982
Len1	3	4.584	4.538	4.602	4.522	4.489	4.481	4.456	4.486
	8	4.843	4.774	4.848	4.737	4.672	4.706	4.639	4.674
	1	4.034	4.056	4.022	4.010	3.969	3.997	3.967	3.957
Len2	3	6.006	5.996	5.979	5.978	5.834	5.858	5.804	5.861
	8	6.321	6.243	6.343	6.218	6.061	6.114	6.067	6.078
Chi	1	2.057	2.205	1.892	1.974	1.839	2.057	2.101	2.141
Var	3	1.462	1.401	1.231	1.303	1.478	1.466	1.538	1.529
	8	0.990	1.097	1.111	1.122	1.174	1.211	1.345	1.314
	1	4.013	4.070	4.004	4.023	4.030	4.084	3.952	3.986
Len1	3	4.689	4.597	4.662	4.607	4.630	4.612	4.554	4.574
	8	4.891	4.771	4.908	4.820	4.749	4.765	4.748	4.750
	1	4.103	4.132	4.101	4.113	4.102	4.133	4.043	4.072
Len2	3	6.309	6.275	6.288	6.275	6.171	6.217	6.142	6.174
	8	6.607	6.487	6.544	6.482	6.378	6.383	6.332	6.353
Exp	1	1.564	1.425	1.820	1.657	1.698	1.624	1.762	1.811
Var	3	1.339	1.256	1.382	1.271	1.771	1.672	1.819	1.736
	8	1.398	1.535	1.404	1.573	1.733	1.914	1.937	1.947
	1	3.951	3.931	3.978	3.928	4.022	4.002	3.888	3.882
Len1	3	4.720	4.593	4.715	4.578	4.712	4.661	4.563	4.541
	8	4.903	4.791	4.914	4.756	4.816	4.848	4.690	4.688
	1	3.952	3.938	4.056	3.995	3.995	4.016	4.007	3.962
Len2	3	6.308	6.249	6.270	6.227	6.210	6.182	6.081	6.132
	8	6.568	6.457	6.660	6.446	6.379	6.381	6.343	6.362
t-dist	1	1.077	1.039	1.141	1.067	1.157	1.122	1.251	1.173
Var	3	0.967	1.097	0.954	0.955	1.140	1.164	1.219	1.157
	8	1.009	1.059	1.038	1.117	1.229	1.236	1.453	1.274
	1	4.042	4.127	4.111	4.118	4.025	4.103	4.017	4.049
Len1	3	4.611	4.556	4.638	4.546	4.554	4.562	4.519	4.513
	8	4.876	4.825	4.885	4.780	4.753	4.803	4.661	4.708
	1	4.120	4.092	4.160	4.141	4.116	4.082	4.104	4.078
Len2	3	6.077	6.064	6.098	6.017	5.951	5.987	5.896	5.914
	8	6.443	6.329	6.422	6.255	6.223	6.218	6.138	6.138

Table B.54. Variance and Lengths of Intervals for Model M9 T=200

	h	PRR	PRRP	KIM	KIMP	PRR%	PRRP%	KIM%	KIMP%
Nor	1	0.980	0.957	1.740	1.250	1.346	1.123	4.221	1.571
Var	3	1.401	1.806	3.368	2.025	2.286	2.436	6.185	2.690
	8	2.832	5.542	10.236	5.407	5.982	5.967	13.566	6.895
	1	4.114	4.082	4.018	4.054	4.060	4.040	3.764	3.984
Len1	3	5.287	5.081	5.189	5.013	5.073	4.995	4.573	4.852
	8	7.122	6.321	7.051	6.273	6.336	6.104	5.529	5.790
	1	4.083	4.072	4.066	4.042	4.034	4.016	3.757	3.989
Len2	3	10.444	10.259	10.287	10.296	10.049	9.960	9.292	9.925
	8	18.181	17.100	17.722	17.454	16.825	16.475	15.370	16.383
Chi	1	2.716	3.404	5.315	3.110	2.443	2.160	6.571	2.534
Var	3	3.768	5.051	9.925	4.207	4.003	3.970	8.951	4.452
	8	3.558	6.248	5.972	4.888	6.351	8.600	10.376	8.805
	1	4.110	4.067	3.979	3.971	4.127	4.088	3.690	3.931
Len1	3	5.409	5.198	5.247	5.093	5.232	5.168	4.604	4.941
	8	7.376	6.641	7.175	6.528	6.645	6.373	5.689	6.046
	1	3.985	3.977	3.972	3.976	3.992	3.981	3.590	3.960
Len2	3	10.359	10.141	10.084	10.205	10.103	10.006	8.915	9.900
	8	18.140	17.227	17.335	17.409	17.004	16.741	15.157	16.402
Exp	1	0.996	2.285	7.274	2.246	2.737	2.574	4.092	2.795
Var	3	2.562	3.595	5.230	2.923	5.011	4.760	11.364	5.879
	8	5.124	8.549	7.949	5.722	9.883	9.296	20.713	12.409
	1	4.088	4.125	3.981	3.980	4.184	4.204	3.694	3.946
Len1	3	5.476	5.196	5.320	5.066	5.420	5.270	4.611	4.932
	8	7.396	6.551	7.256	6.443	6.837	6.519	5.601	5.995
	1	4.060	4.056	4.066	3.941	4.096	4.088	3.711	3.922
Len2	3	10.355	10.209	10.243	10.195	10.248	10.145	9.122	9.942
	8	18.025	17.002	17.440	17.177	17.044	16.756	15.160	16.223
t-dist	1	1.462	1.606	1.768	1.679	1.956	1.712	6.655	2.154
Var	3	2.234	2.826	3.221	2.253	3.585	3.535	10.424	4.313
	8	3.335	5.753	7.508	4.320	6.641	5.929	15.805	9.231
	1	4.144	4.100	4.057	4.112	4.131	4.113	3.769	4.050
Len1	3	5.382	5.135	5.266	5.093	5.194	5.107	4.590	4.940
	8	7.147	6.409	7.091	6.343	6.402	6.177	5.451	5.881
	1	4.198	4.214	4.170	4.185	4.183	4.209	3.801	4.134
Len2	3	10.768	10.604	10.500	10.668	10.516	10.506	9.423	10.343
	8	18.487	17.629	17.727	17.692	17.233	17.149	15.261	16.633

Table B.55. Average Coverage Probability for VARMA(1, 1) T=50

Nor M11	1	2	3	4	5	6	7	8
PRR:	0.8539	0.8440	0.8592	0.8733	0.8874	0.8975	0.9049	0.9113
PRRP:	0.8552	0.8438	0.8581	0.8712	0.8832	0.8913	0.8978	0.9013
KIM:	0.8513	0.8385	0.8530	0.8688	0.8820	0.8922	0.8998	0.9058
KIMP:	0.8591	0.8456	0.8623	0.8744	0.8860	0.8934	0.8995	0.9039
PRR%:	0.8267	0.7959	0.7942	0.7959	0.8031	0.8094	0.8139	0.8166
PRRP%:	0.8264	0.7940	0.7955	0.7979	0.8036	0.8106	0.8155	0.8172
KIM%:	0.8104	0.7787	0.7789	0.7822	0.7870	0.7921	0.7941	0.7951
KIMP%:	0.8221	0.7921	0.7925	0.7928	0.7975	0.8027	0.8082	0.8110
Nor M12	1	2	3	4	5	6	7	8
PRR:	0.7761	0.7921	0.8185	0.8393	0.8557	0.8708	0.8817	0.8890
PRRP:	0.7804	0.7970	0.8254	0.8455	0.8594	0.8717	0.8807	0.8863
KIM:	0.7691	0.7793	0.8068	0.8297	0.8468	0.8608	0.8725	0.8804
KIMP:	0.7897	0.8008	0.8300	0.8522	0.8678	0.8784	0.8869	0.8928
PRR%:	0.7418	0.7315	0.7408	0.7478	0.7524	0.7598	0.7667	0.7705
PRRP%:	0.7362	0.7292	0.7409	0.7488	0.7550	0.7614	0.7668	0.7717
KIM%:	0.7197	0.7077	0.7149	0.7202	0.7268	0.7332	0.7392	0.7425
KIMP%:	0.7319	0.7271	0.7345	0.7403	0.7454	0.7519	0.7583	0.7609
Nor M13	1	2	3	4	5	6	7	8
PRR:	0.8812	0.8740	0.8839	0.8882	0.8917	0.8940	0.8953	0.8952
PRRP:	0.8602	0.8376	0.8362	0.8345	0.8328	0.8290	0.8236	0.8174
KIM:	0.8002	0.7922	0.8058	0.8167	0.8266	0.8335	0.8408	0.8466
KIMP:	0.8539	0.8277	0.8295	0.8312	0.8331	0.8318	0.8318	0.8298
PRR%:	0.8446	0.8139	0.8099	0.8035	0.7956	0.7848	0.7738	0.7603
PRRP%:	0.8364	0.7974	0.7890	0.7810	0.7687	0.7566	0.7424	0.7279
KIM%:	0.6437	0.6256	0.6328	0.6399	0.6438	0.6414	0.6353	0.6276
KIMP%:	0.8116	0.7701	0.7605	0.7531	0.7446	0.7343	0.7223	0.7086
Nor M14	1	2	3	4	5	6	7	8
PRR:	0.8809	0.8788	0.8859	0.8911	0.8934	0.8926	0.8917	0.8913
PRRP:	0.8624	0.8418	0.8392	0.8358	0.8327	0.8262	0.8207	0.8151
KIM:	0.7430	0.7526	0.7438	0.7419	0.7466	0.7522	0.7458	0.7414
KIMP:	0.8575	0.8363	0.8369	0.8361	0.8362	0.8315	0.8288	0.8246
PRR%:	0.8508	0.8277	0.8226	0.8149	0.8070	0.7952	0.7844	0.7732
PRRP%:	0.8335	0.8020	0.7927	0.7818	0.7701	0.7558	0.7407	0.7276
KIM%:	0.5731	0.5833	0.5883	0.5840	0.5834	0.5749	0.5632	0.5523
KIMP%:	0.8180	0.7839	0.7771	0.7679	0.7587	0.7448	0.7324	0.7184

Table B.56. Average Coverage Probability for VARMA(1, 1) T=100

Nor M11	1	2	3	4	5	6	7	8
PRR:	0.8667	0.8530	0.8587	0.8661	0.8731	0.8781	0.8812	0.8841
PRRP:	0.8668	0.8542	0.8617	0.8675	0.8722	0.8762	0.8791	0.8811
KIM:	0.8666	0.8526	0.8613	0.8678	0.8739	0.8796	0.8833	0.8860
KIMP:	0.8676	0.8554	0.8646	0.8702	0.8756	0.8801	0.8825	0.8851
PRR%:	0.8571	0.8307	0.8294	0.8302	0.8325	0.8338	0.8345	0.8357
PRRP%:	0.8550	0.8309	0.8308	0.8314	0.8338	0.8345	0.8353	0.8376
KIM%:	0.8495	0.8266	0.8261	0.8258	0.8259	0.8268	0.8273	0.8287
KIMP%:	0.8538	0.8296	0.8280	0.8293	0.8311	0.8323	0.8336	0.8355

Nor M12	1	2	3	4	5	6	7	8
PRR:	0.7819	0.7896	0.8084	0.8206	0.8286	0.8371	0.8427	0.8469
PRRP:	0.7828	0.7910	0.8115	0.8236	0.8317	0.8377	0.8419	0.8452
KIM:	0.7813	0.7863	0.8053	0.8190	0.8283	0.8371	0.8416	0.8477
KIMP:	0.7883	0.7935	0.8156	0.8280	0.8357	0.8422	0.8466	0.8501
PRR%:	0.7691	0.7639	0.7733	0.7786	0.7806	0.7842	0.7862	0.7864
PRRP%:	0.7656	0.7603	0.7718	0.7768	0.7803	0.7833	0.7852	0.7867
KIM%:	0.7526	0.7505	0.7604	0.7669	0.7695	0.7731	0.7728	0.7750
KIMP%:	0.7619	0.7606	0.7718	0.7764	0.7781	0.7818	0.7840	0.7850

Nor M13	1	2	3	4	5	6	7	8
PRR:	0.8855	0.8731	0.8856	0.8964	0.9037	0.9085	0.9122	0.9149
PRRP:	0.8769	0.8576	0.8637	0.8679	0.8697	0.8704	0.8685	0.8670
KIM:	0.8566	0.8317	0.8377	0.8466	0.8529	0.8578	0.8619	0.8645
KIMP:	0.8727	0.8501	0.8561	0.8603	0.8630	0.8645	0.8640	0.8621
PRR%:	0.8725	0.8471	0.8517	0.8561	0.8575	0.8562	0.8531	0.8505
PRRP%:	0.8665	0.8364	0.8363	0.8361	0.8340	0.8317	0.8268	0.8213
KIM%:	0.7451	0.7227	0.7348	0.7453	0.7525	0.7570	0.7579	0.7579
KIMP%:	0.8612	0.8260	0.8231	0.8219	0.8197	0.8159	0.8109	0.8058

Nor M14	1	2	3	4	5	6	7	8
PRR:	0.8842	0.8759	0.8836	0.8877	0.8895	0.8910	0.8909	0.8915
PRRP:	0.8717	0.8496	0.8467	0.8432	0.8386	0.8346	0.8288	0.8228
KIM:	0.7675	0.7685	0.7786	0.7827	0.7851	0.7828	0.7828	0.7813
KIMP:	0.8681	0.8433	0.8417	0.8407	0.8378	0.8339	0.8310	0.8274
PRR%:	0.8726	0.8516	0.8512	0.8487	0.8446	0.8403	0.8346	0.8303
PRRP%:	0.8642	0.8362	0.8303	0.8258	0.8191	0.8114	0.8049	0.7973
KIM%:	0.5775	0.5977	0.6156	0.6239	0.6286	0.6281	0.6275	0.6245
KIMP%:	0.8545	0.8231	0.8157	0.8092	0.8030	0.7965	0.7888	0.7838

APPENDIX C.

FORTRAN CODE

```
USE GDATA_INT
USE AUTO_MUL_AR_INT
USE WRRRN_INT
IMPLICIT NONE
INTEGER JJ, T, T1
DOUBLE PRECISION Z(50-1,19), ZT(19), ZINV(19,19), ZZ(19,19)
DOUBLE PRECISION XS(2,50-1)
DOUBLE PRECISION MAT5(2,19), MAT6(2,19)
INTEGER JJJ, TT, T2
DOUBLE PRECISION ZZZ(50-1,19), ZTT(19), ZZZZ(19,19)
DOUBLE PRECISION PHIF(2,18)
INTEGER L1, L2, M1
DOUBLE PRECISION SIGMAY(2,18), GAMMA1(2,2), OMEGAF(2,18)
DOUBLE PRECISION BM(19,19)
DOUBLE PRECISION BTT(19,19), GAMMAINV(19,19), IDENTB(19,19)
DOUBLE PRECISION BU(19,19), BTT1(19,19), BU1(19,19), BMT(19,19)
DOUBLE PRECISION TR, TRM(19,19)
INTEGER LOOP2, LOOP3, LOOP4, LOOP5, LOOP6, LOOP7, L3
DOUBLE PRECISION YN(2,50), THETABOOTN(2,19), MAT4(2,19)
INTEGER NN, II, LL, K1, J1
INTEGER UNIFORMN(50-1)
DOUBLE PRECISION DEL, MAXEI, MAT1(2,19), MAT2(2,19), MAT7(2,19)
DOUBLE PRECISION MAT8(18,18), ABSEVAL(18), ABSEVAL1(18)
DOUBLE COMPLEX EVAL(18)
INTEGER I3
DOUBLE PRECISION SIGPOPE(18,18), PIPOPE(18,18), ID(18,18)
DOUBLE PRECISION GAMPOPE(18,18), WT(50-1,2), PH1(18,18)
DOUBLE PRECISION PH2(18,18), PH3(18,18)
DOUBLE PRECISION EVALPOPE(18)
INTEGER LOOPID, LOOPID1
DOUBLE PRECISION SIGPOPEB(18,18), PIPOPEB(18,18), IDB(18,18)
DOUBLE PRECISION GAMPOB(18,18), WTB(50-1,2), PH1B(18,18)
DOUBLE PRECISION PH2B(18,18), PH3B(18,18)
DOUBLE complex EVALPOPEB(18)
INTEGER LOOPIDB, LOOPID1B

INTEGER,parameter:: P0=2, Q=0, N=50, B=999, H=8, IT=100, K=2
INTEGER,parameter:: ALPHA1=25, ALPHA2=975
DOUBLE PRECISION A(K,19), M(K,19), AHAT(K,19), AHATB(K,19)
DOUBLE PRECISION AHATP(K,19), AHATPB(K,19), AHATBOOT(K,19)
DOUBLE PRECISION XT(K,300+N), X(K,N), XBACK(K,N), RT(2,300+N)
DOUBLE PRECISION EPSILON(K,N-1), EPSILONBACK(K,N-1)
DOUBLE PRECISION SIGMA(K,K), GAMMA(19,19), PHI(K,18)
DOUBLE PRECISION BIASKIM(2,19), BIASPRR(2,19)
DOUBLE PRECISION BIASKIMP(2,19), BIASPRRP(2,19)
DOUBLE PRECISION Y(K,N+H), YFUT(K,N+H)
INTEGER LOOPM1, IBOOT
```

```
INTEGER  IRANK, ISEED, LDR, LDRSIG, NOUT, NR, MAXPQ
INTEGER UNIFORM(N-1), UNIFORMBOOT(H), OUTDO, L, J, I, KK, PHIDO, P
DOUBLE PRECISION COV(2,2), R(300+N,2), RSIG(2,2)
DOUBLE PRECISION ABAR(2), FOBS(1000,H,K), IDENT(2,2)
DOUBLE PRECISION YB(B,8,2), YSORT(B), BOUND(8,2,2), RR(8000,2),&
                 RRT(2,8000)
DOUBLE PRECISION CONST, DSUM, YBB(B)
DOUBLE PRECISION COUNT(IT,8), COUNTBACK(IT,8), RES(8)
DOUBLE PRECISION COUNTPER(IT,8), COUNTBACKPER(IT,8), RESPER(8),&
                 RESBACKPER(8), RESBACK(8)
DOUBLE PRECISION COUNTP(IT,8), COUNTBACKP(IT,8), RESP(8),&
                 RESBACKP(8)
DOUBLE PRECISION COUNTPERP(IT,8), COUNTBACKPERP(IT,8),&
                 RESPERP(8), RESBACKPERP(8)
DOUBLE PRECISION SIGMABACK(2,2), SIGMAFOR(2,2), SIGMAPRE(2,16)
DOUBLE PRECISION EPSILONBOOT(2,N-1), EPSILONBOOTT(N-1,2)
DOUBLE PRECISION XBOLD(N,2), THETABOLD(2,3), THETABACKP(2,3)
DOUBLE PRECISION YBACK(N+8,2), YBBBACK(B), YBBACK(B,8,2)
DOUBLE PRECISION YBBACKP(B,8,2), YBP(B,8,2)
DOUBLE PRECISION ZKIM(B,8,2), ZPRR(B,8,2), SIGMAC(2,16)
DOUBLE PRECISION ZKIMP(B,8,2), ZPRRP(B,8,2), SIGMACP(2,16)
DOUBLE PRECISION WPRR(IT,H,3), WPRRP(IT,H,3), WKIM(IT,H,3),&
 WKIMP(IT,H,3), WPRRT(IT,H,3), WPRRTP(IT,H,3), WKIMT(IT,H,3),&
 WKIMTP(IT,H,3), STATWPRR(15,H,3), STATWPRRP(15,H,3),&
 STATWPRRT(15,H,3), STATWPRRTP(15,H,3), STATWKIM(15,H,3),&
 STATWKIMP(15,H,3), STATWKIMT(15,H,3), STATWKIMTP(15,H,3)
DOUBLE PRECISION STATPRR(15,H), STATPRRP(15,H), STATPRRT(15,H),&
 STATPRRTP(15,H), STATKIM(15,H), STATKIMP(15,H), STATKIMT(15,H),&
 STATKIMTP(15,H)
INTEGER NMATRIX, maxlag
REAL(KIND(1E0)) XAIC(N,K), PAR(8,2,2)


COMMON JJ, T, T1
COMMON Z, ZT, ZINV, ZZ
COMMON XS
COMMON MAT5, MAT6
COMMON JJJ, TT, T2
COMMON ZZZ, ZTT, ZZZZ
COMMON PHIF
COMMON L1, L2, M1
COMMON SIGMAY, GAMMA1, OMEGAF
COMMON BM
COMMON BTT, GAMMAINV, IDENTB
COMMON BU, BTT1, BU1, BMT
COMMON TR, TRM
COMMON LOOP2, LOOP3, LOOP4, LOOP5, LOOP6, LOOP7, L3
```

```
      COMMON YN, THETABOOTN, MAT4
      COMMON NN, II, LL, K1, J1
      COMMON UNIFORMN
      COMMON DEL, MAXEI, MAT1, MAT2, MAT7
      COMMON MAT8, ABSEVAL, ABSEVAL1
      COMMON EVAL
      COMMON I3
      COMMON SIGPOPE, PIPOPE, ID
      COMMON GAMPOPE, WT, PH1
      COMMON PH2, PH3
      COMMON EVALPOPE
      COMMON LOOPID, LOOPID1
      COMMON SIGPOPEB, PIPOPEB, IDB
      COMMON GAMPOB, WTB, PH1B
      COMMON PH2B, PH3B
      COMMON EVALPOPEB
      COMMON LOOPIDB, LOOPID1B

      EXTERNAL DLINRG, DSUM, DSVRGN, DRNMVN, DCHFAC, DEVLRG, RNSET,&
               DGEMM
      CALL UMACH (2, NOUT)
      print*, "start change 3"
      open (unit = 3, file = "results50.xls")
      WRITE (3,*) " "
      open (unit = 10, file = "sasar250.txt")
      open (unit = 11, file = "sasar2explain.txt")

  ! Initialize seed of random number generator.
      ISEED = 123457
      CALL RNSET (ISEED)
  ! initializing the time series parameters
      DO PHIDO=1,5
      write(11,*) "loop number", phido
      write(11,*) "number of iterations", IT
      write(11,*) "number of bootstrap itterations", B
      write(11,*) "alpha", alpha1, alpha2
  ! initializing the time series parameters
      COV(1,1) = 1.0D0
      COV(1,2) = 0.3D0
      COV(2,1) = 0.3D0
      COV(2,2) = 1.0D0
      A(:,:)=0.0D0
      IF(PHIDO==1) A(1,2) = 0.9D0
      IF(PHIDO==2) A(1,2) = 0.9D0
      IF(PHIDO==3) A(1,2) = 0.2D0
      IF(PHIDO==4) A(1,2) = 0.4D0
      IF(PHIDO==1) A(1,4) = -0.2D0
```

```
      IF(PHIDO==2) A(1,4) = -0.2D0
      IF(PHIDO==3) A(1,4) = -0.5D0
      IF(PHIDO==4) A(1,4) = 0.45D0
      A(2,2) = 0.5D0
      IF(PHIDO==1) A(2,3) = -0.7D0
      IF(PHIDO==2) A(2,3) = -0.5D0
      IF(PHIDO==3) A(2,3) = -0.5D0
      IF(PHIDO==4) A(2,3) = 1.4D0
      IF(PHIDO==1) A(2,5) = -0.1D0
      IF(PHIDO==2) A(2,5) = -0.125D0
      IF(PHIDO==3) A(2,5) = -0.125D0
      IF(PHIDO==4) A(2,5) = -0.45D0
      A(2,4) = -0.8D0
      M(:,:)=0.0D0
      P = 2
! Start of the bootstrap loop
      DO OUTDO=1,IT
      NR = 300+N
      LDRSIG = 2
      LDR = 300+N
! Obtain the Cholesky factorization.
      CALL DCHFAC (2, COV, 2, 2.2d-14, IRANK, RSIG, 2)
      CALL DRNMVN (NR, K, RSIG, LDRSIG, R, LDR)
      RT=TRANSPOSE(R)
! Generating the time series
      MAXPQ = MAX(P,Q)  ! Start value for none-zero values
      XT(:,:)=0.0D0     ! Setting the time series equal to 0
      DO I=MAXPQ+1, N+300     ! N=# of obs used, 300 obs ommitted
       XT(:,I) = A(:,1)+RT(:,I)
        DO J=1,MAXPQ
         XT(:,I)= XT(:,I)+MATMUL(A(:,K*J-K+2:K*J+1),XT(:,I-J))
         XT(:,I)= XT(:,I)+MATMUL(M(:,K*J-K+1:K*J),RT(:,I-J))
        END DO
      END DO
      X(:,:)=XT(:,301:300+N)       ! Ommiting first 300 observations
!      XAIC=transpose(real(X))     !AIC if p is not specified
!      maxlag=8
!      CALL AUTO_MUL_AR(XAIC,6,NMATRIX,PAR)
!      print*, NMATRIX
      p = 2
      DO I=1, N                    ! Reversing the time series
       XBACK(:,I) = X(:,N-I+1)     ! Used for backward parameter
      END DO                       ! estimation
      AHAT(:,:) = AREST(X,P)       ! Forward parameter estimation
      AHATB(:,:)= AREST(XBACK,P)   ! Backward parameter estimation
      DO I=P+1, N                  ! Finding forward innovations
       EPSILON(:,I-P)=X(:,I)-AHAT(:,1)
```

```
      DO J=1,P
       EPSILON(:,I-P)=EPSILON(:,I-P)-&
                      MATMUL(AHAT(:,K*J-K+2:K*J+1),X(:,I-J))
      END DO !j
    END DO !I
    CONST = 1.0D0/DBLE((N-P-2*P-1))
    SIGMAFOR=CONST*MATMUL(EPSILON(:,1:N-P),&
                   TRANSPOSE(EPSILON(:,1:N-P)))
    GAMMA = GAMMAF(X,P)
    PHI=MAREPEST(AHAT,P)
    SIGMAC=MSEEST2(PHI, AHAT, GAMMA, SIGMAFOR, P)
! centering the residuals following Thombs and Schucany
    ABAR(1) = DSUM(N-P,EPSILON(1,1:N-P),1)/DBLE(N-P)
    ABAR(2) = DSUM(N-P,EPSILON(2,1:N-P),1)/DBLE(N-P)
    CONST = SQRT((DBLE(N-P))/(DBLE(N-P-2*P-1)))  ! Constant
    DO I=1, N-P
     EPSILON(1,I)=(EPSILON(1,I)-ABAR(1))*CONST
     EPSILON(2,I)=(EPSILON(2,I)-ABAR(2))*CONST
    END DO !I
    DO I=1, N-P             ! find backward inovations
     EPSILONBACK(:,I)=X(:,I)-AHATB(:,1)
      DO J=1,P
       EPSILONBACK(:,I)=EPSILONBACK(:,I)-&
                      MATMUL(AHATB(:,K*J-K+2:K*J+1),X(:,I+J))
      END DO !J
    END DO !I
! centering the residuals following Thombs and Schucany
    ABAR(1) = DSUM(N-P,EPSILONBACK(1,1:N-P),1)/DBLE(N-P)
    ABAR(2) = DSUM(N-P,EPSILONBACK(2,1:N-P),1)/DBLE(N-P)
    CONST =SQRT((DBLE(N-P))/(DBLE(N-P-2*P-1)))       ! Constant
    DO I=1, N-P
     EPSILONBACK(1,I)=(EPSILONBACK(1,I)-ABAR(1))*CONST ! Standard
     EPSILONBACK(2,I)=(EPSILONBACK(2,I)-ABAR(2))*CONST ! Standard
    END DO !I
! Estimation of parameters and bias correction
    BIASPRR(:,:) = FBIAS(EPSILON, AHAT, X(:,1:18), P)
    BIASPRRP(:,:) = -1.0D0/DBLE(N)*POPE(X, EPSILON, AHAT, P)
    AHATP(:,:)= BIASCORR(AHAT, BIASPRRP, P)
    AHAT(:,:)= BIASCORR(AHAT, BIASPRR, P)
    BIASKIM(:,:) = FBIAS(EPSILONBACK, AHATB, XBACK(:,1:18), P)
    BIASKIMP(:,:) = -1.0D0/DBLE(50)*&
                   POPEB(XBACK, EPSILONBACK, AHATB, P)
    AHATPB(:,:)= BIASCORR(AHATB, BIASKIMP, P)
    AHATB(:,:)= BIASCORR(AHATB, BIASKIM, P)
    YFUT(:,1:N)=X(:,:) ! Estimating future values
    DO I=N+1, N+H
     YFUT(:,I) = AHAT(:,1)
```

```
        DO J=1, P
         YFUT(:,I)= YFUT(:,I)+MATMUL(AHAT(:,K*J-K+2:K*J+1),YFUT(:,I-J))
        END DO
      END DO
! Bootstrap intervals forward Kilian bias correction
      DO IBOOT=1, B                  ! do loop bootstrapping y_n+h
       Y(:,1:P) = X(:,1:P)           ! initializing bootstrap series
       CALL RNUND (N-P,N-P,UNIFORM(1:N-P))
       DO I=P+1, N                   ! do loop for generating BTTS
       Y(:,I) = AHAT(:,1)+EPSILON(:,UNIFORM(I-P))
        DO J=1, P
         Y(:,I)= Y(:,I)+MATMUL(AHAT(:,K*J-K+2:K*J+1),Y(:,I-J))
        END DO
       END DO                   ! end do loop for generating BTTS
        AHATBOOT = AREST(Y(:,1:N), P)
        AHATBOOT = BIASCORR(AHATBOOT, BIASPRR, P)
        DO I=P+1, N                 ! Finding forward innovations
         EPSILONBOOT(:,I-P)=Y(:,I)-AHATBOOT(:,1)
         DO J=1,P
          EPSILONBOOT(:,I-P)=EPSILONBOOT(:,I-P)-&
                      MATMUL(AHATBOOT(:,K*J-K+2:K*J+1),Y(:,I-J))
         END DO !j
        END DO !I
! centering the residuals following Thombs Schucany
       ABAR(1) = DSUM(N-P,EPSILONBOOT(1,1:N-P),1)/DBLE(N-P)
       ABAR(2) = DSUM(N-P,EPSILONBOOT(2,1:N-P),1)/DBLE(N-P)
       CONST =SQRT((DBLE(N-P))/(DBLE(N-P-2*P-1)))
        DO J=1, N-P
         EPSILONBOOT(1,J)=(EPSILONBOOT(1,J)-ABAR(1))*CONST
         EPSILONBOOT(2,J)=(EPSILONBOOT(2,J)-ABAR(2))*CONST
        END DO !J
! creating the percentile t-percentile points
        PHI=MAREPEST(AHATBOOT, P)
        GAMMA=GAMMAF(Y(:,1:N), P)
        EPSILONBOOTT(:,:)=transpose(EPSILONBOOT(:,:))
        SIGMA=1.0D0/DBLE(N-P-2*P-1)*MATMUL(EPSILONBOOT(:,1:N-P)&
              ,EPSILONBOOTT(1:N-P,:))
        SIGMAPRE=MSEEST2(PHI, AHATBOOT, GAMMA, SIGMA, P)
       CALL RNUND (H,N-P,UNIFORMBOOT)
       Y(:,N-P:N)=X(:,N-P:N)
       DO I=N+1, N+H
        Y(:,I) = AHATBOOT(:,1)+EPSILON(:,UNIFORMBOOT(I-N))
         DO J=1, P
          Y(:,I)= Y(:,I)+MATMUL(AHATBOOT(:,K*J-K+2:K*J+1),Y(:,I-J))
         END DO
         YB(IBOOT,I-N,1) = Y(1,I)
         YB(IBOOT,I-N,2) = Y(2,I)
```

```
        END DO !I
        DO J=1,H
          ZPRR(IBOOT,J,1) = (Y(1,N+J)-YFUT(1,J+N))/&
            SQRT((SIGMAPRE(1,2*(J-1)+1)))
          ZPRR(IBOOT,J,2) = (Y(2,N+J)-YFUT(2,J+N))/&
            SQRT((SIGMAPRE(2,2*(J-1)+2)))
        END DO !J
       Y(:,:)=0.0D0 ! just to be safe
       END DO !IBOOT
! Bootstrap intervals forward Nicholls and Pope bias correction!
      DO IBOOT=1, B                  ! do loop bootstrapping y_n+1
       Y(:,1:P) = X(:,1:P)           ! initializing bootstrap series
       CALL RNUND (N-P,N-P,UNIFORM(1:N-P))
       DO I=P+1, N                   ! do loop for generating BTTS
       Y(:,I) = AHATP(:,1)+EPSILON(:,UNIFORM(I-P))
        DO J=1, P
         Y(:,I)= Y(:,I)+MATMUL(AHATP(:,K*J-K+2:K*J+1),Y(:,I-J))
        END DO
       END DO                      ! end do loop for generating BTTS
        AHATBOOT(:,:) = AREST(Y(:,1:N), P)
        DO I=P+1, N                 ! Finding forward innovations
         EPSILONBOOT(:,I-P)=Y(:,I)-AHATBOOT(:,1)
          DO J=1,P
           EPSILONBOOT(:,I-P)=EPSILONBOOT(:,I-P)-&
                      MATMUL(AHATBOOT(:,K*J-K+2:K*J+1),Y(:,I-J))
          END DO !j
        END DO !I
! centering the residuals following Schrucany
       ABAR(1) = DSUM(N-P,EPSILONBOOT(1,1:N-P),1)/DBLE(N-P)
       ABAR(2) = DSUM(N-P,EPSILONBOOT(2,1:N-P),1)/DBLE(N-P)
       CONST =SQRT((DBLE(N-P))/(DBLE(N-P-2*P-1)))
        DO J=1, N-P
         EPSILONBOOT(1,J)=(EPSILONBOOT(1,J)-ABAR(1))*CONST
         EPSILONBOOT(2,J)=(EPSILONBOOT(2,J)-ABAR(2))*CONST
        END DO !J
       BIASPRRP(:,:) = -1.0D0/DBLE(N)*POPE(Y(:,1:N), EPSILONBOOT,&
                                          AHATBOOT, P)
       AHATBOOT(:,:)=BIASCORR(AHATBOOT, BIASPRRP, P)
! creating the percentile t-percentile points
       PHI=MAREPEST(AHATBOOT, P)
       GAMMA=GAMMAF(Y(:,1:N), P)
       EPSILONBOOTT(:,:)=transpose(EPSILONBOOT(:,:))
       SIGMA=1.0D0/DBLE(N-P-2*P-1)*MATMUL(EPSILONBOOT(:,1:N-P),&
             EPSILONBOOTT(1:N-P,:))
       SIGMAPRE=MSEEST2(PHI, AHATBOOT, GAMMA, SIGMA, P)
       CALL RNUND (H,N-P,UNIFORMBOOT)
       Y(:,N-P:N)=X(:,N-P:N)
```

```
      DO I=N+1, N+H
       Y(:,I) = AHATBOOT(:,1)+EPSILON(:,UNIFORMBOOT(I-N))
        DO J=1, P
         Y(:,I)= Y(:,I)+MATMUL(AHATBOOT(:,K*J-K+2:K*J+1),Y(:,I-J))
        END DO
       YBP(IBOOT,I-N,1) = Y(1,I)
       YBP(IBOOT,I-N,2) = Y(2,I)
      END DO !I
      DO J=1,H
       ZPRRP(IBOOT,J,1) = (Y(1,N+J)-YFUT(1,J+N))/&
         SQRT((SIGMAPRE(1,2*(J-1)+1)))
       ZPRRP(IBOOT,J,2) = (Y(2,N+J)-YFUT(2,J+N))/&
         SQRT((SIGMAPRE(2,2*(J-1)+2)))
       END DO !J
      Y(:,:)=0.0D0 ! just to be safe
     END DO !IBOOT
! Bootstrap intervals backward Kilian bias correction!
     DO IBOOT=1, B                    ! do loop bootstrapping y_n+1
      Y(:,N-P:N)=X(:,N-P:N)           ! initializing bootstrap series
      CALL RNUND (N-P,N-P,UNIFORM(1:N-P))
      DO I=P+1, N               ! do loop for generating BTTS
      Y(:,N-I) = AHATB(:,1)+EPSILONBACK(:,UNIFORM(I-P))
       DO J=1, P
        Y(:,N-I+1)= Y(:,N-I+1)+&
        MATMUL(AHATB(:,K*J-K+2:K*J+1),Y(:,N-I+1+J))
       END DO
      END DO                    ! end do loop for generating BTTS
      AHATBOOT(:,:) = AREST(Y(:,1:N), P)
      AHATBOOT(:,:)=BIASCORR(AHATBOOT, BIASPRR, P)
      DO I=P+1, N                ! Finding forward innovations
       EPSILONBOOT(:,I-P)=Y(:,I)-AHATBOOT(:,1)
        DO J=1,P
         EPSILONBOOT(:,I-P)=EPSILONBOOT(:,I-P)-&
                     MATMUL(AHATBOOT(:,K*J-K+2:K*J+1),Y(:,I-J))
        END DO !j
       END DO !I
! centering the residuals following Schrucany
      ABAR(1) = DSUM(N-P,EPSILONBOOT(1,1:N-P),1)/DBLE(N-P) ! aver.
      ABAR(2) = DSUM(N-P,EPSILONBOOT(2,1:N-P),1)/DBLE(N-P) ! aver.
      CONST =SQRT((DBLE(N-P))/(DBLE(N-P-2*P-1)))       ! Constant
       DO J=1, N-P
        EPSILONBOOT(1,J)=(EPSILONBOOT(1,J)-ABAR(1))*CONST
        EPSILONBOOT(2,J)=(EPSILONBOOT(2,J)-ABAR(2))*CONST
       END DO !J
! creating the percentile t-percentile points
       PHI=MAREPEST(AHATBOOT, P)
       GAMMA=GAMMAF(Y(:,1:N), P)
```

```
       EPSILONBOOTT(:,:)=transpose(EPSILONBOOT(:,:))
       SIGMA=1.0D0/DBLE(N-P-2*P-1)*MATMUL(EPSILONBOOT(:,1:N-P),&
                EPSILONBOOTT(1:N-P,:))
       SIGMAPRE=MSEEST2(PHI, AHATBOOT, GAMMA, SIGMA, P)
      CALL RNUND (H,N-P,UNIFORMBOOT)
      DO I=N+1, N+H
       Y(:,I) = AHATBOOT(:,1)+EPSILON(:,UNIFORMBOOT(I-N))
        DO J=1, P
         Y(:,I)= Y(:,I)+MATMUL(AHATBOOT(:,K*J-K+2:K*J+1),Y(:,I-J))
        END DO
        YBBACK(IBOOT,I-N,1) = Y(1,I)
        YBBACK(IBOOT,I-N,2) = Y(2,I)
       END DO !I
       DO J=1,H
         ZKIM(IBOOT,J,1) = (Y(1,N+J)-YFUT(1,J+N))/&
           SQRT((SIGMAPRE(1,2*(J-1)+1)))
         ZKIM(IBOOT,J,2) = (Y(2,N+J)-YFUT(2,J+N))/&
           SQRT((SIGMAPRE(2,2*(J-1)+2)))
       END DO !J
      Y(:,:)=0.0D0 ! just to be safe
      END DO !IBOOT
! Bootstrap intervals backward Nicholls and Pope bias correction!
     DO IBOOT=1, B                 ! do loop bootstrapping y_n+1
      Y(:,N-P:N)=X(:,N-P:N)        ! initializing bootstrap series
      CALL RNUND (N-P,N-P,UNIFORM(1:N-P))
      DO I=P+1, N                  ! do loop for generating BTTS
      Y(:,N-I+1) = AHATPB(:,1)+EPSILONBACK(:,UNIFORM(I-P))
       DO J=1, P
        Y(:,N-I+1)= Y(:,N-I+1)+&
                    MATMUL(AHATPB(:,K*J-K+2:K*J+1),Y(:,N-I+1+J))
       END DO
      END DO                   ! end do loop for generating BTTS
       AHATBOOT(:,:) = AREST(Y(:,1:N), P)
       DO I=P+1, N                 ! Finding forward innovations
        EPSILONBOOT(:,I-P)=Y(:,I)-AHATBOOT(:,1)
         DO J=1,P
          EPSILONBOOT(:,I-P)=EPSILONBOOT(:,I-P)-&
                     MATMUL(AHATBOOT(:,K*J-K+2:K*J+1),Y(:,I-J))
         END DO !j
       END DO !I
! centering the residuals following Schrucany
      ABAR(1) = DSUM(N-P,EPSILONBOOT(1,1:N-P),1)/DBLE(N-P) ! aver.
      ABAR(2) = DSUM(N-P,EPSILONBOOT(2,1:N-P),1)/DBLE(N-P) ! aver.
      CONST =SQRT((DBLE(N-P))/(DBLE(N-P-2*P-1)))       ! Constant
       DO J=1, N-P
        EPSILONBOOT(1,J)=(EPSILONBOOT(1,J)-ABAR(1))*CONST
        EPSILONBOOT(2,J)=(EPSILONBOOT(2,J)-ABAR(2))*CONST
```

```
        END DO !J
        BIASPRRP(:,:) = -1.0D0/DBLE(N)*POPE(Y(:,1:N), EPSILONBOOT,&
                                               AHATBOOT, P)
        AHATBOOT(:,:)=BIASCORR(AHATBOOT, BIASPRRP, P)
! creating the percentile t-percentile points
        PHI=MAREPEST(AHATBOOT, P)
        GAMMA=GAMMAF(Y(:,1:N), P)
        EPSILONBOOTT(:,:)=transpose(EPSILONBOOT(:,:))
        SIGMA=1.0D0/DBLE(N-P-2*P-1)*MATMUL(EPSILONBOOT(:,1:N-P),&
                   EPSILONBOOTT(1:N-P,:))
        SIGMAPRE=MSEEST2(PHI, AHATBOOT, GAMMA, SIGMA, P)
       CALL RNUND (H,N-P,UNIFORMBOOT)
       DO I=N+1, N+H
        Y(:,I) = AHATBOOT(:,1)+EPSILON(:,UNIFORMBOOT(I-N))
         DO J=1, P
          Y(:,I)= Y(:,I)+MATMUL(AHATBOOT(:,K*J-K+2:K*J+1),Y(:,I-J))
         END DO
         YBBACKP(IBOOT,I-N,1) = Y(1,I)
         YBBACKP(IBOOT,I-N,2) = Y(2,I)
        END DO !I
        DO J=1,H
          ZKIMP(IBOOT,J,1) = (Y(1,N+J)-YFUT(1,J+N))/&
            SQRT((SIGMAPRE(1,2*(J-1)+1)))
          ZKIMP(IBOOT,J,2) = (Y(2,N+J)-YFUT(2,J+N))/&
            SQRT((SIGMAPRE(2,2*(J-1)+2)))
        END DO !J
       Y(:,:)=0.0D0 ! just to be safe
       END DO !IBOOT
      NR = 8000
      LDRSIG = 2
      LDR = 8000
      CALL DRNMVN (NR, K, RSIG, LDRSIG, RR, LDR)
      RRT=TRANSPOSE(RR)
      YFUT(:,1:N)=X(:,1:N)
      DO IBOOT=1, 1000
       DO I=N+1, N+H
        YFUT(:,I) = A(:,1)+RRT(:,(IBOOT-1)*H+I-N)
         DO J=1, P
          YFUT(:,I)= YFUT(:,I)+MATMUL(A(:,K*J-K+2:K*J+1),YFUT(:,I-J))
         END DO
       FOBS(IBOOT,I-N,1)=YFUT(1,I)
       FOBS(IBOOT,I-N,2)=YFUT(2,I)
       END DO
      END DO
      YFUT(:,1:N)=X(:,:)
      DO I=N+1, N+H
       YFUT(:,I) = AHAT(:,1)
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```
       DO J=1, P
        YFUT(:,I)= YFUT(:,I)+MATMUL(AHAT(:,K*J-K+2:K*J+1),YFUT(:,I-J))
       END DO
     END DO
! Bonferoni intervals Forward Kilian bias correction
     DO J=1,H
      DO I=1,K
        YBB=YB(:,J,I)
        CALL DSVRGN(B,YBB,YSORT)
        BOUND(J,I,1)=YSORT(alpha1)
        BOUND(J,I,2)=YSORT(alpha2)
      END DO !I
     WPRR(OUTDO,J,1)=ABS(BOUND(J,1,2)-BOUND(J,1,1))
     WPRR(OUTDO,J,2)=ABS(BOUND(J,2,2)-BOUND(J,2,1))
     WPRR(OUTDO,J,3)=ABS((BOUND(J,2,2)-BOUND(J,2,1))*&
                    (BOUND(J,1,2)-BOUND(J,1,1)))
     END DO !J
     COUNT(OUTDO,:)=0.0D0
     DO J=1,H
     DO I=1,1000
     IF(((BOUND(J,1,1)<=FOBS(I,J,1)).AND.&
                        (FOBS(I,J,1)<=BOUND(J,1,2)) ).AND.&
       ((BOUND(J,2,1)<=FOBS(I,J,2)) .AND. &
                         (FOBS(I,J,2)<=BOUND(J,2,2)))) THEN
         COUNT(OUTDO,J)=COUNT(OUTDO,J)+1.0D0
      END IF
     END DO !I
     END DO !J
! Bonferoni intervals Forward Nicholls and Pope bias correction
     DO J=1,8
      DO I=1,2
        YBB=YBP(:,J,I)
        CALL DSVRGN(B,YBB,YSORT)
        BOUND(J,I,1)=YSORT(alpha1)
        BOUND(J,I,2)=YSORT(alpha2)
      END DO !I
     WPRRP(OUTDO,J,1)=ABS(BOUND(J,1,2)-BOUND(J,1,1))
     WPRRP(OUTDO,J,2)=ABS(BOUND(J,2,2)-BOUND(J,2,1))
     WPRRP(OUTDO,J,3)=ABS((BOUND(J,2,2)-BOUND(J,2,1))*&
                    (BOUND(J,1,2)-BOUND(J,1,1)))
     END DO !J
     COUNTP(OUTDO,:)=0.0D0
     DO J=1,8
     DO I=1,1000
     IF(((BOUND(J,1,1)<=FOBS(I,J,1)).AND.&
                        (FOBS(I,J,1)<=BOUND(J,1,2)) ).AND.&
       ((BOUND(J,2,1)<= FOBS(I,J,2)) .AND. &
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                    (FOBS(I,J,2)<=BOUND(J,2,2)))) THEN
          COUNTP(OUTDO,J)=COUNTP(OUTDO,J)+1.0D0
       END IF
      END DO !I
      END DO !J
! percentile t-intervals Forward Kilian bias correction
      DO J=1,8
       DO I=1,2
        YBB=ZPRR(:,J,I)
        CALL DSVRGN(B,YBB,YSORT)
        BOUND(J,I,1)=YFUT(I,J+N)-YSORT(alpha2)*sqrt(SIGMAC(I,2*(J-1)+I))
        BOUND(J,I,2)=YFUT(I,J+N)-YSORT(alpha1)*sqrt(SIGMAC(I,2*(J-1)+I))
       END DO !I
      WPRRT(OUTDO,J,1)=ABS(BOUND(J,1,2)-BOUND(J,1,1))
      WPRRT(OUTDO,J,2)=ABS(BOUND(J,2,2)-BOUND(J,2,1))
      WPRRT(OUTDO,J,3)=ABS((BOUND(J,2,2)-BOUND(J,2,1))*&
                     (BOUND(J,1,2)-BOUND(J,1,1)))
      END DO !J
     COUNTPER(OUTDO,:)=0.0D0
     DO J=1,8
      DO I=1,1000
      IF(((BOUND(J,1,1)<=FOBS(I,J,1)).AND.&
                    (FOBS(I,J,1)<=BOUND(J,1,2)) ).AND.&
        ((BOUND(J,2,1)<=FOBS(I,J,2)) .AND. &
                   (FOBS(I,J,2)<=BOUND(J,2,2)))) THEN
          COUNTPER(OUTDO,J)=COUNTPER(OUTDO,J)+1.0D0
       END IF
      END DO !I
      END DO !J
! percentile-t intervals Forward Nicholls and Pope bias correction
      DO J=1,8
       DO I=1,2
       YBB=ZPRRP(:,J,I)
       CALL DSVRGN(B,YBB,YSORT)
       BOUND(J,I,1)=YFUT(I,J+N)-YSORT(alpha2)*sqrt(SIGMAC(I,2*(J-1)+I))
       BOUND(J,I,2)=YFUT(I,J+N)-YSORT(alpha1)*sqrt(SIGMAC(I,2*(J-1)+I))
      END DO !I
      WPRRTP(OUTDO,J,1)=ABS(BOUND(J,1,2)-BOUND(J,1,1))
      WPRRTP(OUTDO,J,2)=ABS(BOUND(J,2,2)-BOUND(J,2,1))
      WPRRTP(OUTDO,J,3)=ABS((BOUND(J,2,2)-BOUND(J,2,1))*&
                     (BOUND(J,1,2)-BOUND(J,1,1)))
      END DO !J
      COUNTPERP(OUTDO,:)=0.0D0
      DO J=1,8
      DO I=1,1000
      IF(((BOUND(J,1,1)<=FOBS(I,J,1)).AND.&
                    (FOBS(I,J,1)<=BOUND(J,1,2)) ).AND.&
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  ((BOUND(J,2,1)<=FOBS(I,J,2)) .AND. &
                  (FOBS(I,J,2)<=BOUND(J,2,2)))) THEN
    COUNTPERP(OUTDO,J)=COUNTPERP(OUTDO,J)+1.0D0
 END IF
END DO !I
END DO !J
! Bonferoni intervals backward Kilian bias correction
BOUND=0.0d0
DO J=1,8
 DO I=1,2
   YBBBACK=YBBACK(:,J,I)
   CALL DSVRGN(B,YBBBACK,YSORT)
   BOUND(J,I,1)=YSORT(alpha1)
   BOUND(J,I,2)=YSORT(alpha2)
 END DO !I
WKIM(OUTDO,J,1)=ABS(BOUND(J,1,2)-BOUND(J,1,1))
WKIM(OUTDO,J,2)=ABS(BOUND(J,2,2)-BOUND(J,2,1))
WKIM(OUTDO,J,3)=ABS((BOUND(J,2,2)-BOUND(J,2,1))*&
              (BOUND(J,1,2)-BOUND(J,1,1)))
END DO !J
COUNTBACK(OUTDO,:)=0.0D0
DO J=1,8
DO I=1,1000
IF(((BOUND(J,1,1)<=FOBS(I,J,1)).AND.&
                  (FOBS(I,J,1)<=BOUND(J,1,2)) ).AND.&
  ((BOUND(J,2,1)<=FOBS(I,J,2)) .AND. &
                  (FOBS(I,J,2)<=BOUND(J,2,2)))) THEN
    COUNTBACK(OUTDO,J)=COUNTBACK(OUTDO,J)+1.0D0
 END IF
END DO !I
END DO !J
! Bonferoni intervals backward Nicholls and Pope bias correction
BOUND=0.0d0
DO J=1,8
 DO I=1,2
   YBBBACK=YBBACKP(:,J,I)
   CALL DSVRGN(B,YBBBACK,YSORT)
   BOUND(J,I,1)=YSORT(alpha1)
   BOUND(J,I,2)=YSORT(alpha2)
 END DO !I

WKIMP(OUTDO,J,1)=ABS(BOUND(J,1,2)-BOUND(J,1,1))
WKIMP(OUTDO,J,2)=ABS(BOUND(J,2,2)-BOUND(J,2,1))
WKIMP(OUTDO,J,3)=ABS((BOUND(J,2,2)-BOUND(J,2,1))*&
              (BOUND(J,1,2)-BOUND(J,1,1)))
END DO !J
COUNTBACKP(OUTDO,:)=0.0D0
```

```
      DO J=1,8
      DO I=1,1000
      IF(((BOUND(J,1,1)<=FOBS(I,J,1)).AND.&
                         (FOBS(I,J,1)<=BOUND(J,1,2)) ).AND.&
        ((BOUND(J,2,1)<=FOBS(I,J,2)) .AND. &
                        (FOBS(I,J,2)<=BOUND(J,2,2)))) THEN
          COUNTBACKP(OUTDO,J)=COUNTBACKP(OUTDO,J)+1.0D0
       END IF
      END DO !I
      END DO !J
! percentile-t intervals backward Kilian bias correction
      DO J=1,8
       DO I=1,2
       YBB=ZKIM(:,J,I)
       CALL DSVRGN(B,YBB,YSORT)
       BOUND(J,I,1)=YFUT(I,J+N)-YSORT(alpha2)*sqrt(SIGMAC(I,2*(J-1)+I))
       BOUND(J,I,2)=YFUT(I,J+N)-YSORT(alpha1)*sqrt(SIGMAC(I,2*(J-1)+I))
       END DO !I
      WKIMT(OUTDO,J,1)=ABS(BOUND(J,1,2)-BOUND(J,1,1))
      WKIMT(OUTDO,J,2)=ABS(BOUND(J,2,2)-BOUND(J,2,1))
      WKIMT(OUTDO,J,3)=ABS((BOUND(J,2,2)-BOUND(J,2,1))*&
                     (BOUND(J,1,2)-BOUND(J,1,1)))
      END DO !J
      COUNTBACKPER(OUTDO,:)=0.0D0
      DO J=1,8
      DO I=1,1000
      IF(((BOUND(J,1,1)<=FOBS(I,J,1)).AND.&
                         (FOBS(I,J,1)<=BOUND(J,1,2)) ).AND.&
        ((BOUND(J,2,1)<=FOBS(I,J,2)) .AND. &
                        (FOBS(I,J,2)<=BOUND(J,2,2)))) THEN
          COUNTBACKPER(OUTDO,J)=COUNTBACKPER(OUTDO,J)+1.0D0
       END IF
      END DO !I
      END DO !J

! percentile-t intervals backward Nicholls and Pope bias correction
      DO J=1,8
       DO I=1,2
       YBB=ZKIMP(:,J,I)
       CALL DSVRGN(B,YBB,YSORT)
       BOUND(J,I,1)=YFUT(I,J+N)-YSORT(alpha2)*sqrt(SIGMAC(I,2*(J-1)+I))
       BOUND(J,I,2)=YFUT(I,J+N)-YSORT(alpha1)*sqrt(SIGMAC(I,2*(J-1)+I))
      END DO !I
      WKIMTP(OUTDO,J,1)=ABS(BOUND(J,1,2)-BOUND(J,1,1))
      WKIMTP(OUTDO,J,2)=ABS(BOUND(J,2,2)-BOUND(J,2,1))
      WKIMTP(OUTDO,J,3)=ABS((BOUND(J,2,2)-BOUND(J,2,1))*&
                     (BOUND(J,1,2)-BOUND(J,1,1)))
```

```
END DO !J
COUNTBACKPERP(OUTDO,:)=0.0D0
DO J=1,8
DO I=1,1000
IF(((BOUND(J,1,1)<=FOBS(I,J,1)).AND.&
                  (FOBS(I,J,1)<=BOUND(J,1,2)) ).AND.&
  ((BOUND(J,2,1)<= FOBS(I,J,2)) .AND.&
                  (FOBS(I,J,2)<=BOUND(J,2,2)))) THEN
    COUNTBACKPERP(OUTDO,J)=COUNTBACKPERP(OUTDO,J)+1.0D0
 END IF
END DO !I
END DO !J
END DO !OUTDO
CALL DUVSTA (0, IT, H, COUNT, IT, 0, 0, 0, 0.95,&
             0.95, 0, STATPRR, 15, 0)
CALL DUVSTA (0, IT, H, COUNTBACK, IT, 0, 0, 0, 0.95,&
             0.95, 0, STATKIM, 15, 0)
CALL DUVSTA (0, IT, H, COUNTPER, IT, 0, 0, 0, 0.95,&
             0.95, 0, STATPRRT, 15, 0)
CALL DUVSTA (0, IT, H, COUNTBACKPER, IT, 0, 0, 0, 0.95,&
             0.95, 0, STATKIMT, 15, 0)
CALL DUVSTA (0, IT, H, COUNTP, IT, 0, 0, 0, 0.95,&
             0.95, 0, STATPRRP, 15, 0)
CALL DUVSTA (0, IT, H, COUNTBACKP, IT, 0, 0, 0, 0.95,&
             0.95, 0, STATKIMP, 15, 0)
CALL DUVSTA (0, IT, H, COUNTPERP, IT, 0, 0, 0, 0.95,&
             0.95, 0, STATPRRTP, 15, 0)
CALL DUVSTA (0, IT, H, COUNTBACKPERP, IT, 0, 0, 0, 0.95,&
             0.95, 0, STATKIMTP, 15, 0)
DO J=1,3
CALL DUVSTA (0, IT, 8, WPRR(:,:,J), IT, 0, 0, 0, 0.95,&
             0.95, 0, STATWPRR(:,:,J), 15, 0)
CALL DUVSTA (0, IT, 8, WKIM(:,:,J), IT, 0, 0, 0, 0.95,&
             0.95, 0, STATWKIM(:,:,J), 15, 0)
CALL DUVSTA (0, IT, 8, WPRRT(:,:,J), IT, 0, 0, 0, 0.95,&
             0.95, 0, STATWPRRT(:,:,J), 15, 0)
CALL DUVSTA (0, IT, 8, WKIMT(:,:,J), IT, 0, 0, 0, 0.95,&
             0.95, 0, STATWKIMT(:,:,J), 15, 0)
CALL DUVSTA (0, IT, 8, WPRRP(:,:,J), IT, 0, 0, 0, 0.95,&
             0.95, 0, STATWPRRP(:,:,J), 15, 0)
CALL DUVSTA (0, IT, 8, WKIMP(:,:,J), IT, 0, 0, 0, 0.95,&
             0.95, 0, STATWKIMP(:,:,J), 15, 0)
CALL DUVSTA (0, IT, 8, WPRRTP(:,:,J), IT, 0, 0, 0, 0.95,&
             0.95, 0, STATWPRRTP(:,:,J), 15, 0)
CALL DUVSTA (0, IT, 8, WKIMTP(:,:,J), IT, 0, 0, 0, 0.95,&
             0.95, 0, STATWKIMTP(:,:,J), 15, 0)
END DO !J
```

```
      WRITE (3,10000) ((A(I,J),J=1,P+1),I=1,K)
10000 FORMAT (' PHI: ', /,  (1X,3F7.4))
      WRITE (3,00001) (STATPRR(1,I)/1000D0,I=1,8)
00001 FORMAT (' PRR: ', (1X,8F10.4))
      WRITE (3,00002) (STATPRRP(1,I)/1000D0,I=1,8)
00002 FORMAT (' PRRP: ', (1X,8F10.4))
      WRITE (3,00003) (STATKIM(1,I)/1000D0,I=1,8)
00003 FORMAT (' KIM: ', (1X,8F10.4))
      WRITE (3,00004) (STATKIMP(1,I)/1000D0,I=1,8)
00004 FORMAT (' KIMP: ', (1X,8F10.4))
      WRITE (3,00005) (STATPRRT(1,I)/1000D0,I=1,8)
00005 FORMAT (' PRR%: ', (1X,8F10.4))
      WRITE (3,00006) (STATPRRTP(1,I)/1000D0,I=1,8)
00006 FORMAT (' PRRP%: ', (1X,8F10.4))
      WRITE (3,00007) (STATKIMT(1,I)/1000D0,I=1,8)
00007 FORMAT (' KIM%: ', (1X,8F10.4))
      WRITE (3,00008) (STATKIMTP(1,I)/1000D0,I=1,8)
00008 FORMAT (' KIMP%: ',(1X,8F10.4))
      WRITE (3,*) "VAR"
      WRITE (3,00011) (STATPRR(2,I)/1000D0,I=1,8)
00011 FORMAT (' PRR: ', (1X,8F10.4))
      WRITE (3,00012) (STATPRRP(2,I)/1000D0,I=1,8)
00012 FORMAT (' PRRP: ', (1X,8F10.4))
      WRITE (3,00013) (STATKIM(2,I)/1000D0,I=1,8)
00013 FORMAT (' KIM: ', (1X,8F10.4))
      WRITE (3,00014) (STATKIMP(2,I)/1000D0,I=1,8)
00014 FORMAT (' KIMP: ', (1X,8F10.4))
      WRITE (3,00015) (STATPRRT(2,I)/1000D0,I=1,8)
00015 FORMAT (' PRR%: ', (1X,8F10.4))
      WRITE (3,00016) (STATPRRTP(2,I)/1000D0,I=1,8)
00016 FORMAT (' PRRP%: ', (1X,8F10.4))
      WRITE (3,00017) (STATKIMT(2,I)/1000D0,I=1,8)
00017 FORMAT (' KIM%: ', (1X,8F10.4))
      WRITE (3,00018) (STATKIMTP(2,I)/1000D0,I=1,8)
00018 FORMAT (' KIMP%: ',(1X,8F10.4))

      WRITE (3,*) "Area"
      DO J=1,3
      WRITE (3,00021) (STATWPRR(1,I,J),I=1,8)
00021 FORMAT (' PRR: ', (1X,8F10.4))
      WRITE (3,00022) (STATWPRRP(1,I,J),I=1,8)
00022 FORMAT (' PRRP: ', (1X,8F10.4))
      WRITE (3,00023) (STATWKIM(1,I,J),I=1,8)
00023 FORMAT (' KIM: ', (1X,8F10.4))
      WRITE (3,00024) (STATWKIMP(1,I,J),I=1,8)
00024 FORMAT (' KIMP: ', (1X,8F10.4))
      WRITE (3,00025) (STATWPRRT(1,I,J),I=1,8)
```

```
00025 FORMAT (' PRR%: ', (1X,8F10.4))
      WRITE (3,00026) (STATWPRRTP(1,I,J),I=1,8)
00026 FORMAT (' PRRP%: ', (1X,8F10.4))
      WRITE (3,00027) (STATWKIMT(1,I,J),I=1,8)
00027 FORMAT (' KIM%: ', (1X,8F10.4))
      WRITE (3,00028) (STATWKIMTP(1,I,J),I=1,8)
00028 FORMAT (' KIMP%: ',(1X,8F10.4))
      END DO !J
      DO I=1,H
      WRITE (10,*) PHIDO, I,&
                    STATPRR(1,I), STATPRRP(1,I), STATKIM(1,I),&
                    STATKIMP(1,I), STATPRRT(1,I), STATPRRTP(1,I),&
                    STATKIMT(1,I), STATKIMTP(1,I),&
                    STATPRR(2,I), STATPRRP(2,I), STATKIM(2,I),&
                    STATKIMP(2,I), STATPRRT(2,I), STATPRRTP(2,I),&
                    STATKIMT(2,I), STATKIMTP(2,I),&
         STATWPRR(1,I,1), STATWPRRP(1,I,1), STATWKIM(1,I,1),&
         STATWKIMP(1,I,1), STATWPRRT(1,I,1), STATWPRRTP(1,I,1),&
         STATWKIMT(1,I,1), STATWKIMTP(1,I,1),&
         STATWPRR(1,I,2), STATWPRRP(1,I,2), STATWKIM(1,I,2),&
         STATWKIMP(1,I,2), STATWPRRT(1,I,2), STATWPRRTP(1,I,2),&
         STATWKIMT(1,I,2), STATWKIMTP(1,I,2),&
         STATWPRR(1,I,3), STATWPRRP(1,I,3), STATWKIM(1,I,3),&
         STATWKIMP(1,I,3), STATWPRRT(1,I,3), STATWPRRTP(1,I,3),&
         STATWKIMT(1,I,3), STATWKIMTP(1,I,3)
      END DO !I
      END DO ! PHIDO
      close(3)
      close(10)
      close(11)
      CONTAINS
! Functions: need to copy the headers that are given at the begining
! of the program at the begining of each of the functions.
      FUNCTION AREST(XMATRIX, P1) RESULT(MATPHI)
! Function finds least squares estimators
      DOUBLE PRECISION:: XMATRIX(2,50)
      INTEGER:: P1
      DOUBLE PRECISION:: MATPHI(2,19)
      EXTERNAL DLINRG, DSUM, DSVRGN, DRNMVN, DCHFAC, DEVLRG, RNSET,&
               DGEMM
      DO T= P1, 50-1
       ZT(1)=1.0d0
        DO T1=0,P1-1
         DO JJ=1, 2
          ZT(1+JJ+2*T1)=XMATRIX(JJ,T-T1)
         END DO !JJ
       END DO ! T1
```

```
      Z(T-P1+1,:)=ZT(:)
      END DO !T
      MAT6=0.0d0
      CALL DGEMM ('N', 'N', 1+2*P1, 1+2*P1, 50-P1, 1.0D0, &
              TRANSPOSE(Z(1:50-P1,1:2*P1+1)),1+2*P1,&
                        Z(1:50-P1,1:2*P1+1),&
              50-P1, 0.0D0, ZZ(1:1+2*P1,1:1+2*P1), 1+2*P1)
      CALL DLINRG(1+2*P1,ZZ(1:1+2*P1,1:1+2*P1),1+2*P1,&
                ZINV(1:1+2*P1,1:1+2*P1),1+2*P1)
      XS=XMATRIX(:,P1+1:50)
      CALL DGEMM('N', 'N', 2, 1+2*P1, 50-P1, 1.0D0, XS(:,1:50-P1),&
            2,Z(1:50-P1,1:2*P1+1),50-P1, 0.0D0, MAT5(:,1:2*P1+1), 2)
      CALL DGEMM ('N', 'N', 2, 1+2*P1, 1+2*P1, 1.0D0, MAT5(:,1:2*P1+1),&
                 2, ZINV(1:1+2*P1,1:1+2*P1),&
                  1+2*P1, 0.0D0, MAT6(:,1:1+2*P1), 2)
      MATPHI(:,:)=MAT6(:,:)
      END FUNCTION AREST

      FUNCTION GAMMAF(XXMATRIX, P2) RESULT(GAMMAF1)
! Function finds Gamma
      DOUBLE PRECISION:: XXMATRIX(2,50)
      INTEGER:: P2
      DOUBLE PRECISION:: GAMMAF1(19,19)
      EXTERNAL DLINRG, DSUM, DSVRGN, DRNMVN, DCHFAC, DEVLRG, RNSET,&
              DGEMM
      DO TT= P2, 50-1
       ZTT(1)=1.0d0
        DO T2=0,P2-1
         DO JJJ=1, 2
          ZTT(1+JJJ+2*T2)=XXMATRIX(JJJ,TT-T2)
         END DO !JJJ
       END DO ! T2
      ZZZ(TT-P2+1,:)=ZTT(:)
      END DO !TT
      ZZZZ=0.0D0
      CALL DGEMM ('N', 'N', 1+2*P2, 1+2*P2, 50-P2, 1.0D0,&
      TRANSPOSE(ZZZ(1:50-P2,1:2*P2+1)),1+2*P2, ZZZ(1:50-P2,1:2*P2+1),&
               50-P2, 0.0D0, ZZZZ(1:2*P2+1,1:2*P2+1), 1+2*P2)
      GAMMAF1=1.0D0/50.D0*ZZZZ
      END FUNCTION GAMMAF

      FUNCTION MAREPEST(AMAT, P3) RESULT(PHIRE) !only k=2!
! Function finds parameters of VMA of VAR
      DOUBLE PRECISION:: AMAT(2,19)
      INTEGER:: P3
      DOUBLE PRECISION:: PHIRE(2,18)
      EXTERNAL DLINRG, DSUM, DSVRGN, DRNMVN, DCHFAC, DEVLRG, RNSET,&
```

```
          DGEMM
PHIF(:,:)=0.0D0
PHIF(1,1)=1.0D0
PHIF(2,2)=1.0D0
PHIF(:,3:4)=AMAT(:,2:3)
DO L1=3,9 ! phi2 correspondes with L1=3
 M1= MIN(L1,P3)
 PHIF(:,(2*L1-1):(2*L1))=0.0D0
 DO L2=1,M1
  PHIF(:,(2*L1-1):(2*L1))=PHIF(:,(2*L1-1):(2*L1))+&
  MATMUL(PHIF(:,2*(L1-L2)-1:2*(L1-L2)),&
         AMAT(:,2*(L2-1)+2:2*(L2-1)+3))
 END DO !L2
END DO !L1
PHIRE(:,:)=PHIF(:,:)
END FUNCTION MAREPEST

FUNCTION MSEEST2(PHIF1, AMAT1, GAMMA2, SIGMA2, P3) RESULT(SIGMARE)
! Function finds estimate of Sigma_Y(h)
DOUBLE PRECISION:: PHIF1(2,18), AMAT1(2,19), GAMMA2(19,19),&
                   SIGMA2(2,2)
INTEGER:: P3
DOUBLE PRECISION:: SIGMARE(2,16)
EXTERNAL DLINRG, DSUM, DSVRGN, DRNMVN, DCHFAC, DEVLRG, RNSET,&
         DGEMM
BM(:,:) = 0.0D0
BM(1,1) = 1.0D0
BM(2:3,:) = AMAT1(:,:)
DO L3=4,19
 BM(L3,L3-2)=1.0D0
END DO !L3
IDENTB = 0.0D0
DO L3=1,19
IDENTB(L3,L3) = 1.0D0
END DO !L3
SIGMAY=0.0D0
DO L3=1,8 ! phi2 correspondes with loop1=3
 SIGMAY(:,(2*L3+1):(2*L3+2))=&
     SIGMAY(:,(2*L3-1):(2*L3))+&
     MATMUL(PHIF1(:,2*L3-1:2*L3),&
     MATMUL(sigma2,TRANSPOSE(PHIF1(:,2*L3-1:2*L3))))
END DO !L3
OMEGAF(:,:)=0.0D0
SIGMAY(:,1:16)=SIGMAY(:,3:18)
CALL DLINRG(2*P3+1,GAMMA2(1:2*P3+1,1:2*P3+1),2*P3+1,&
            GAMMAINV(1:2*P3+1,1:2*P3+1),2*P3+1)
DO LOOP4=1,8
```

```
 DO LOOP5=0,LOOP4-1
  DO LOOP6=0,LOOP4-1
    IF(LOOP4-1-LOOP5==0) THEN
     BTT(1:2*P3+1,1:2*P3+1)=IDENTB(1:2*P3+1,1:2*P3+1)
    END IF
    IF(LOOP4-1-LOOP5==1) THEN
     BTT(1:2*P3+1,1:2*P3+1)=TRANSPOSE(BM(1:2*P3+1,1:2*P3+1))
    END IF
    IF(LOOP4-1-LOOP5>=2) THEN
    BTT(1:2*P3+1,1:2*P3+1)=TRANSPOSE(BM(1:2*P3+1,1:2*P3+1))
    DO LOOP7=2, LOOP4-1-LOOP5
     BMT(1:2*P3+1,1:2*P3+1)=TRANSPOSE(BM(1:2*P3+1,1:2*P3+1))
     BTT1=MATMUL(BTT(1:2*P3+1,1:2*P3+1),BMT(1:2*P3+1,1:2*P3+1))
     BTT(1:2*P3+1,1:2*P3+1)=BTT1(1:2*P3+1,1:2*P3+1)
    END DO !LOOP7
    END IF
    IF(LOOP4-1-LOOP6==0) THEN
     BU(1:2*P3+1,1:2*P3+1)=IDENTB(1:2*P3+1,1:2*P3+1)
    END IF
    IF(LOOP4-1-LOOP6==1) THEN
     BU(1:2*P3+1,1:2*P3+1)=BM(1:2*P3+1,1:2*P3+1)
    END IF
    IF(LOOP4-1-LOOP6>=2) THEN
    BU(1:2*P3+1,1:2*P3+1) = BM(1:2*P3+1,1:2*P3+1)
    DO LOOP7=2,LOOP4-1-LOOP6
     BU1=MATMUL(BU(1:2*P3+1,1:2*P3+1),BM(1:2*P3+1,1:2*P3+1))
     BU(1:2*P3+1,1:2*P3+1)=BU1(1:2*P3+1,1:2*P3+1)
    END DO ! LOOP8
    END IF
   BTT1=MATMUL(BTT(1:2*P3+1,1:2*P3+1),&
                        GAMMAINV(1:2*P3+1,1:2*P3+1))
   BU1=MATMUL(BU(1:2*P3+1,1:2*P3+1),&
                       GAMMA2(1:2*P3+1,1:2*P3+1))
   TRM=MATMUL(BTT1(1:2*P3+1,1:2*P3+1),&
                       BU1(1:2*P3+1,1:2*P3+1))
   TR=0.0d0
   DO L3=1, 2*P3+1
   TR=TRM(L3,L3)+TR
   END DO !L3
OMEGAF(:,(2*LOOP4-1):(2*LOOP4))=OMEGAF(:,(2*LOOP4-1):(2*LOOP4))+&
     TR*MATMUL(PHIF1(:,2*(LOOP5+1)-1:2*(LOOP5+1)),&
     MATMUL(SIGMA2,TRANSPOSE(PHIF1(:,2*(LOOP6+1)-1:2*(LOOP6+1)))))
  END DO !LOOP6
 END DO !LOOP5
END DO !LOOP4
SIGMARE= SIGMAY(:,1:16)+1.0D0/50.0D0*OMEGAF(:,1:16)
END FUNCTION MSEEST2
```

```
      FUNCTION FBIAS(INV, EST, START, P4) RESULT(PHIBIAS)
! Function finds Kilian bias
      DOUBLE PRECISION:: INV(2,50-1), EST(2,19), START(2,18)
      INTEGER:: P4
      DOUBLE PRECISION:: PHIBIAS(2,19)
      EXTERNAL DLINRG, DSUM, DSVRGN, DRNMVN, DCHFAC, DEVLRG, RNSET,&
               DGEMM
      K1=2
      NN=50
      MAT4(:,:)=0.0d0
       DO II=1, 1000             ! do loop bootstrapping y_n
       YN(:,:)=0.0D0 ! just to be safe
        YN=START(:,1:2*P4)
        CALL RNUND (NN-P4,NN-P4,UNIFORMN(1:NN-P4))
        DO LL=P4+1, NN              ! do loop for generating BTS
         YN(:,LL) = EST(:,1)+INV(:,UNIFORMN(LL-P4))
        DO J1=1,P4
         YN(:,LL)= YN(:,LL)+&
                   MATMUL(EST(:,K1*J1-K1+2:K1*J1+1),YN(:,LL-J1))
        END DO
        END DO                    ! end do loop  for generating BTS
        THETABOOTN(:,:) = AREST(YN, P4)
        MAT4(:,:)=MAT4(:,:)+THETABOOTN(:,:)
       END DO !II
      PHIBIAS(:,:)=MAT4(:,:)/1000.0d0-EST(:,:)
      END FUNCTION FBIAS

      FUNCTION BIASCORR(THETACOR, BIASCOR, P5) RESULT(MAT3)
! Function corrects for bias ensuring stationarity
      DOUBLE PRECISION:: THETACOR(2,19), BIASCOR(2,19)
      INTEGER:: P5
      DOUBLE PRECISION:: MAT3(2,19)
      EXTERNAL DLINRG, DSUM, DSVRGN, DRNMVN, DCHFAC, DEVLRG, RNSET,&
               DGEMM
      DEL=1.0d0
      MAT7(:,:) = BIASCOR(:,:)
      MAT1(:,:) = THETACOR(:,:)
      MAT2(:,:) = MAT1(:,:)
      MAT8=0.0D0
      MAT8(1:2,:)=THETACOR(:,2:19)
      DO I3=1,18-2
       MAT8(I3+2,I3)=1.0D0
      END DO !I3
      CALL DEVLRG (2*P5, MAT8(1:2*P5,1:2*P5), 2*P5, EVAL(1:2*P5))
      ABSEVAL(1:2*P5)= ABS(EVAL(1:2*P5))
      CALL DSVRGN(2*P5,ABSEVAL(1:2*P5),ABSEVAL1(1:2*P5))
```

```
      MAXEI = ABSEVAL1(2*P5)
      IF ( MAXEI < 1.0D0) THEN
          MAT2(:,:) = MAT1(:,:) - MAT7(:,:)
          MAT8=0.0D0
          MAT8(1:2,:)=MAT2(:,2:19)
          DO I3=1,18-2
           MAT8(I3+2,I3)=1.0D0
          END DO !I3
          CALL DEVLRG (2*P5, MAT8(1:2*P5,1:2*P5), 2*P5, EVAL(1:2*P5))
          ABSEVAL(1:2*P5)= ABS(EVAL(1:2*P5))
          CALL DSVRGN(2*p5,ABSEVAL(1:2*P5),ABSEVAL1(1:2*P5))
          MAXEI = ABSEVAL1(2*P5)
        IF ( MAXEI >= 1.0D0) THEN
         DO
          IF(MAXEI < 1.0D0) EXIT
          if(DEL<-100.0d0) exit
          DEL = DEL-0.01D0
          if(del==0.0d0) then
          DEL = DEL-0.01D0
          end if
          MAT7(:,:)=DEL*MAT7(:,:)
          MAT2(:,:) = MAT1(:,:) - MAT7(:,:)
          MAT8=0.0D0
          MAT8(1:2,:)=MAT2(:,2:19)
          DO I3=1,18-2
           MAT8(I3+2,I3)=1.0D0
          END DO !I3
          CALL DEVLRG (2*P5, MAT8(1:2*P5,1:2*P5), 2*P5, EVAL(1:2*P5))
          ABSEVAL(1:2*P5)= ABS(EVAL(1:2*P5))
          CALL DSVRGN(2*p5,ABSEVAL(1:2*P5),ABSEVAL1(1:2*P5))
          MAXEI = ABSEVAL1(2*P5)
         END DO
        END IF
      END IF
      MAT3(:,:)=MAT2(:,:)
      END FUNCTION BIASCORR

      FUNCTION POPE(XM, EPS, THE, P6) RESULT(POPEM)
! Function finds bias estimate following Nicholls and Pope
      DOUBLE PRECISION:: XM(2,50), EPS(2,49), THE(2,19)
      INTEGER:: P6
      DOUBLE PRECISION:: POPEM(2,19)
      EXTERNAL DLINRG, DSUM, DSVRGN, DRNMVN, DCHFAC, DEVLRG, RNSET,&
               DGEMM
      SIGPOPE=0.0D0
      SIGPOPE(1:2,1:2)=1.0D0/DBLE(50-P6)*&
                       MATMUL(EPS(:,1:50-P6),TRANSPOSE(EPS(:,1:50-P6)))
```

```
PIPOPE=0.0D0
PIPOPE(1:2,:)=THE(:,2:19)
DO LOOPID=1,18-2
 PIPOPE(2+LOOPID,LOOPID)=1.0D0
END DO !LOOPID
ID=0.0D0
DO LOOPID=1,18
 ID(LOOPID,LOOPID)=1.0D0
END DO !LOOPID
DO LOOPID=1+P6,50
 DO LOOPID1=1, P6
  WT(LOOPID-P6,2*LOOPID1-1)=XM(1,LOOPID-LOOPID1)
  WT(LOOPID-P6,2*LOOPID1)=XM(2,LOOPID-LOOPID1)
 END DO !LOOPID1
END DO !LOOPID
GAMPOPE=1.0D0/50.0D0*MATMUL(TRANSPOSE(WT(1:50-P6,1:2*P6)),&
        WT(1:50-P6,1:2*P6))
PH1=0.0D0
PH1=ID(1:2*P6,1:2*P6)-TRANSPOSE(PIPOPE(1:2*P6,1:2*P6))
CALL DLINRG(2*P6,PH1(1:2*P6,1:2*P6),2*P6,&
             PH2(1:2*P6,1:2*P6),2*P6)
PH1=ID(1:2*P6,1:2*P6)-MATMUL(TRANSPOSE(PIPOPE(1:2*P6,1:2*P6)),&
    TRANSPOSE(PIPOPE(1:2*P6,1:2*P6)))
CALL DLINRG(2*P6,PH1(1:2*P6,1:2*P6),2*P6,&
             PH3(1:2*P6,1:2*P6),2*P6)
PH2=TRANSPOSE(PIPOPE(1:2*P6,1:2*P6))
PH1=MATMUL(PH2(1:2*P6,1:2*P6),PH3(1:2*P6,1:2*P6))
PH3=PH1(1:2*P6,1:2*P6)+PH2(1:2*P6,1:2*P6)
CALL DEVLRG (2*P6, PIPOPE(1:2*P6,1:2*P6),2*P6, EVALPOPE(1:2*P6))
DO LOOPID=1,2*P6
 PH1=ID(1:2*P6,1:2*P6)-ABS(EVALPOPE(LOOPID))*&
     TRANSPOSE(PIPOPE(1:2*P6,1:2*P6))
CALL DLINRG(2*P6,PH1(1:2*P6,1:2*P6),2*P6,&
             PH2(1:2*P6,1:2*P6),2*P6)
 PH1=ABS(EVALPOPE(LOOPID))*PH2(1:2*P6,1:2*P6)
 PH3=PH3(1:2*P6,1:2*P6)+PH1(1:2*P6,1:2*P6)
END DO !LOOPID
PH1=MATMUL(SIGPOPE(1:2*P6,1:2*P6),PH3(1:2*P6,1:2*P6))
CALL DLINRG(2*P6,GAMPOPE(1:2*P6,1:2*P6),2*P6,&
             PH3(1:2*P6,1:2*P6),2*P6)
PH2=MATMUL(PH1(1:2*P6,1:2*P6),PH3(1:2*P6,1:2*P6))
POPEM=0.0D0
POPEM(:,2:2*P6+1)=PH2(1:2,1:2*P6)
END FUNCTION POPE

FUNCTION POPEB(XMB, EPSB, THEB, P7) RESULT(POPEMB)
! Function finds bias correction based on Nicholls and Pope backward
```

```
DOUBLE PRECISION:: XMB(2,50), EPSB(2,49), THEB(2,19)
INTEGER:: P7
DOUBLE PRECISION:: POPEMB(2,19)
EXTERNAL DLINRG, DSUM, DSVRGN, DRNMVN, DCHFAC, DEVLRG, RNSET,&
         DGEMM

SIGPOPEB=0.0D0
SIGPOPEB(1:2,1:2)=1.0D0/DBLE(50-P7)*&
   MATMUL(EPSB(:,1:50-P7),TRANSPOSE(EPSB(:,1:50-P7)))
PIPOPEB=0.0D0
PIPOPEB(1:2,:)=THEB(:,2:19)
DO LOOPIDB=1,18-2
 PIPOPEB(2+LOOPIDB,LOOPIDB)=1.0D0
END DO !LOOPID
IDB=0.0D0
DO LOOPIDB=1,18
 IDB(LOOPIDB,LOOPIDB)=1.0D0
END DO !LOOPIDB
DO LOOPIDB=1+P7,50
 DO LOOPID1B=1, P7
  WTB(LOOPIDB-P7,2*LOOPID1B-1)=XMB(1,LOOPIDB-LOOPID1B)
  WTB(LOOPIDB-P7,2*LOOPID1B)=XMB(2,LOOPIDB-LOOPID1B)
 END DO !LOOPID1B
END DO !LOOPIDB
GAMPOB=1.0D0/50.0D0*MATMUL(TRANSPOSE(WTB(1:50-P7,1:2*P7)),&
        WTB(1:50-P7,1:2*P7))
PH1B=0.0D0
PH1B=IDB(1:2*P7,1:2*P7)-TRANSPOSE(PIPOPEB(1:2*P7,1:2*P7))
CALL DLINRG(2*P7,PH1B(1:2*P7,1:2*P7),2*P7,&
             PH2B(1:2*P7,1:2*P7),2*P7)
PH1B=IDB(1:2*P7,1:2*P7)-MATMUL(TRANSPOSE(PIPOPEB(1:2*P7,1:2*P7)),&
    TRANSPOSE(PIPOPEB(1:2*P7,1:2*P7)))
CALL DLINRG(2*P7,PH1B(1:2*P7,1:2*P7),2*P7,&
             PH3B(1:2*P7,1:2*P7),2*P7)
PH2B=TRANSPOSE(PIPOPEB(1:2*P7,1:2*P7))
PH1B=MATMUL(PH2B(1:2*P7,1:2*P7),PH3B(1:2*P7,1:2*P7))
PH3B=PH1B(1:2*P7,1:2*P7)+PH2B(1:2*P7,1:2*P7)
CALL DEVLRG (2*P7, PIPOPEB(1:2*P7,1:2*P7),2*P7, EVALPOPEB(1:2*P7))
DO LOOPIDB=1,2*P7
 PH1B=IDB(1:2*P7,1:2*P7)-ABS(EVALPOPEB(LOOPIDB))*&
     TRANSPOSE(PIPOPEB(1:2*P7,1:2*P7))
CALL DLINRG(2*P7,PH1B(1:2*P7,1:2*P7),2*P7,&
             PH2B(1:2*P7,1:2*P7),2*P7)
 PH1B=ABS(EVALPOPEB(LOOPIDB))*PH2B(1:2*P7,1:2*P7)
 PH3B=PH3B(1:2*P7,1:2*P7)+PH1B(1:2*P7,1:2*P7)
END DO !LOOPIDB
PH1B=MATMUL(SIGPOPEB(1:2*P7,1:2*P7),PH3B(1:2*P7,1:2*P7))
```

```
      CALL DLINRG(2*P7,GAMPOB(1:2*P7,1:2*P7),2*P7,&
                  PH3B(1:2*P7,1:2*P7),2*P7)
      PH2B=MATMUL(PH1B(1:2*P7,1:2*P7),PH3B(1:2*P7,1:2*P7))
      POPEMB=0.0D0
      POPEMB(:,2:2*P7+1)=PH2B(1:2,1:2*P7)
      END FUNCTION POPEB
     END PROGRAM
! source 'which cttsetup.csh'
! $F90 -o ar2 $F90FLAGS tintar2.f90 $LINK_F90
```

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VITA

Florian Sebastian Rueck was born April 20, 1977, in Heidenheim, Germany. His primary education was conducted in Wisconsin, and his secondary education in Germany. He graduated in June 1996 with the Abitur from the Waldorfschule, Heidenheim. After finishing his mandatory one-year term in alternative military service, he enrolled at the University of Ulm, where he received his Bachelor of Science degree in Mathematics with a focus on Economics in 1999. He then started working towards his Masters in Science at Ulm. In 2001, he came to the University of Missouri - Rolla as an exchange student, where he received his Master of Science degree in Applied Mathematics in 2003. He then enrolled in the Ph.D. program in Mathematics and Statistics at the University of Missouri - Rolla. During this time he has also been working as a Graduate Teaching Assistant. After completing his Ph.D. he will begin working for Capital One in Richmond, Virgina.